Chapter 10 Conics

10-1 Introduction to Analytic Geometry

Pages 619–620 Check for Understanding

1. negative distances have no meaning

2. Use the distance formula to show that the measure of the distance from the midpoint to either endpoint is the same.

3a. Yes; the distance from B to A is $\frac{a\sqrt{5}}{2}$ and the distance from B to C is also $\frac{a\sqrt{5}}{2}$.

3b. Yes; the distance from B to A is $\sqrt{a^2 + b^2}$ and the distance from C to A is also $\sqrt{a^2 + b^2}$.

3c. No; the distance from A to B is $\sqrt{a^2 + b^2}$, the distance from A to C is $a + b$, and the distance from B to C is $b\sqrt{2}$.

4. (1) Show that two pairs of opposite sides are parallel by showing that slopes of the lines through each pair of opposite sides are equal.
(2) Show that two pairs of opposite sides are congruent by showing that the distance between the vertices forming each pair of opposite sides are equal.
(3) Show that one pair of opposite sides is parallel and congruent by showing that the slopes of the lines through that pair of sides are equal and that the distances between the endpoints of each pair of segments are equal.
(4) Show that the diagonals bisect each other by showing that the midpoints of the diagonals coincide.

5. $d = \sqrt{(x_2 - x_1)^2 + (y^2 - y_1)^2}$
$d = \sqrt{(5 - 5)^2 + (11 - 1)^2}$
$d = \sqrt{0^2 + 10^2}$
$d = \sqrt{100}$ or 10
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{5 + 5}{2}, \frac{1 + 11}{2}\right)$
$= (5, 6)$

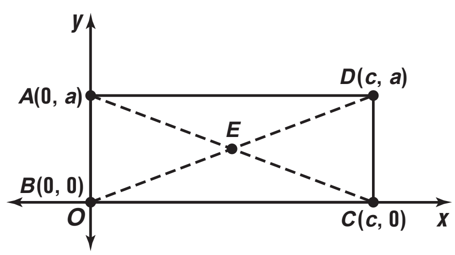
6. $d = \sqrt{(x_2 - x_2)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(-4 - 0)^2 + (-3 - 0)^2}$
$d = \sqrt{(-4)^2 + (-3)^2}$
$d = \sqrt{25}$ or 5
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + (-4)}{2}, \frac{0 + (-3)}{2}\right)$
$= (-2, -1.5)$

7. $d = \sqrt{[0 - (-2)]^2 + (4 - 2)^2}$
$d = \sqrt{2^2 + 2^2}$
$d = \sqrt{8}$ or $2\sqrt{2}$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 0}{2}, \frac{2 + 4}{2}\right)$
$= (-1, 3)$

8. $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(6 - 3)^2 + (2 - 4)^2}$
$= \sqrt{13}$
slope of $\overline{AB}$
$m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{2 - 4}{6 - 3}$ or $-\frac{2}{3}$
$DC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(8 - 5)^2 + (7 - 9)^2}$
$= \sqrt{13}$
slope of $\overline{DC}$
$m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{7 - 9}{8 - 5}$ or $-\frac{2}{3}$
Yes; $\overline{AB} \cong \overline{DC}$ since $AB = \sqrt{13}$ and $DC = \sqrt{13}$, and $\overline{AB} \parallel \overline{DC}$ since the slope of $\overline{AB}$ is $-\frac{2}{3}$ and the slope of $\overline{DC}$ is also $-\frac{2}{3}$.

9. $XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[-1 - (-3)]^2 + (-6 - 2)^2}$
$= \sqrt{68}$ or $2\sqrt{17}$
$XZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[5 - (-3)]^2 + (0 - 2)^2}$
$= \sqrt{68}$ or $2\sqrt{17}$
Yes; $\overline{XY} \cong \overline{XZ}$, since $XY = 2\sqrt{17}$ and $XZ = 2\sqrt{17}$, therefore $\triangle XYZ$ is isosceles.

10a.



10b. $BD = \sqrt{(c - 0)^2 + (a - 0)^2}$
$= \sqrt{c^2 + a^2}$
$AC = \sqrt{(c - 0)^2 + (0 - a)^2}$
$= \sqrt{c^2 + a^2}$
Thus, $\overline{AC} \cong \overline{BD}$.

10c. The midpoint of $\overline{AC}$ is $\left(\frac{c}{2}, \frac{a}{2}\right)$. The midpoint of $\overline{BD}$ is $\left(\frac{c}{2}, \frac{a}{2}\right)$. Therefore, the diagonals intersect at their common midpoint, $E\left(\frac{c}{2}, \frac{a}{2}\right)$. Thus, $\overline{AE} \cong \overline{EC}$ and $\overline{BE} \cong \overline{ED}$.

10d. The diagonals of a rectangle are congruent and bisect each other.

11a. Both players are located along a diagonal of the field with endpoints (0, 0) and (80, 120). The kicker's teammate is located at the midpoint of this diagonal.
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + 80}{2}, \frac{0 + 120}{2}\right)$
$= (40, 60)$

11b. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(40 - 0)^2 + (60 - 0)^2}$
$d = \sqrt{40^2 + 60^2}$
$d = \sqrt{5200}$
$d = 20\sqrt{13}$ or about 72 yards

Pages 620–622 Exercises

12. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{[4 - (-1)]^2 + (13 - 1)^2}$
$d = \sqrt{5^2 + 12^2}$
$d = \sqrt{169}$ or 13
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-1 + 4}{2}, \frac{1 + 13}{2}\right)$
$= (1.5, 7)$

13. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(-1 - 1)^2 + (-3 - 3)^2}$
$d = \sqrt{(-2)^2 + (-6)^2}$
$d = \sqrt{40}$ or $2\sqrt{10}$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1 + (-1)}{2}, \frac{3 + (-3)}{2}\right)$
$= (0, 0)$

14. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(0 - 8)^2 + (8 - 0)^2}$
$d = \sqrt{(-8)^2 + 8^2}$
$d = \sqrt{128}$ or $8\sqrt{2}$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{8 + 0}{2}, \frac{8 + 0}{2}\right)$
$= (4, 4)$

15. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{[5 - (-1)]^2 + [-3 - (-6)]^2}$
$d = \sqrt{6^2 + 3^2}$
$d = \sqrt{45}$ or $3\sqrt{5}$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-1 + 5}{2}, \frac{-6 + (-3)}{2}\right)$
$= (2, -4.5)$

16. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(7\sqrt{2} - 3\sqrt{2})^2 + [-1 - (-5)]^2}$
$d = \sqrt{(4\sqrt{2})^2 + 4^2}$
$d = \sqrt{48}$ or $4\sqrt{3}$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3\sqrt{2} + 7\sqrt{2}}{2}, \frac{-5 + (-1)}{2}\right)$
$= (5\sqrt{2}, -3)$

17. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(a - a)^2 + (-9 - 7)^2}$
$d = \sqrt{0^2 + (-16)^2}$
$d = \sqrt{256}$ or 16
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{a + a}{2}, \frac{7 + (-9)}{2}\right)$
$= (a, -1)$

18. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{[r - 2 - (6 + r)]^2 + (s - s)^2}$
$d = \sqrt{(-8)^2 + 0^2}$
$d = \sqrt{64}$ or 8
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{6 + r + r - 2}{2}, \frac{s + s}{2}\right)$
$= \left(\frac{2r + 4}{2}, \frac{2s}{2}\right)$
$= (r + 2, s)$

19. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(c + 2 - c)^2 + (d - 1 - d)^2}$
$d = \sqrt{2^2 + (-1)^2}$
$d = \sqrt{5}$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{c + c + 2}{2}, \frac{d + d - 1}{2}\right)$
$= \left(\frac{2c + 2}{2}, \frac{2d - 1}{2}\right)$
$= \left(c + 1, d - \frac{1}{2}\right)$

20. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{[w - (w - 2)]^2 + (4w - w)^2}$
$d = \sqrt{2^2 + (3w)^2}$
$d = \sqrt{4 + 9w^2}$ or $\sqrt{9w^2 + 4}$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{w - 2 + w}{2}, \frac{w + 4w}{2}\right)$
$= \left(w - 1, \frac{5}{2}w\right)$

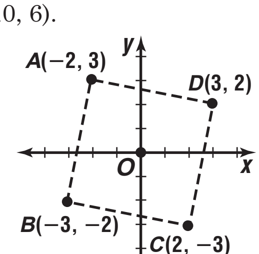
21. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$20 = \sqrt{(-2a - a)^2 + [7 - (-9)]^2}$
$20 = \sqrt{(-3a)^2 + 16^2}$
$20 = \sqrt{9a^2 + 256}$
$400 = 9a^2 + 256$
$144 = 9a^2$
$a^2 = 16$
$a = \pm\sqrt{16}$ or ± 4

22. Let D have coordinates (x_2, y_2).
$\left(\frac{4 + x_2}{2}, \frac{-1 + y_2}{2}\right) = \left(-3, \frac{5}{2}\right)$

$\frac{4 + x_2}{2} = -3$ $\qquad$ $\frac{-1 + y_2}{2} = \frac{5}{2}$
$4 + x_2 = -6$ $\qquad$ $-1 + y_2 = 5$
$x_2 = -10$ $\qquad$ $y_2 = 6$

Then D has coordinates $(-10, 6)$.

23. Let the vertices of the quadrilateral be $A(-2, 3)$, $B(-2, -3)$, $C(2, -3)$, and $D(3, 2)$.



A quadrilateral is a parallelogram if one pair of opposite sides are parallel and congruent.
$\overline{AD}$ and $\overline{BC}$ are one pair of opposite sides.

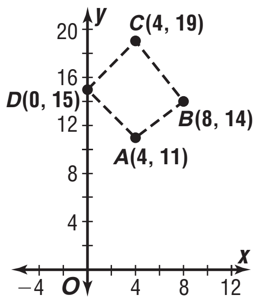
slope of $\overline{AD}$
$m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{2 - 3}{3 - (-2)}$
$= -\frac{1}{5}$

slope of $\overline{BC}$
$m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-3 - (-2)}{2 - (-3)}$
$= -\frac{1}{5}$

Their slopes are equal, therefore $\overline{AD} \parallel \overline{BC}$.
$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[3 - (-2)]^2 + (2 - 3)^2}$
$= \sqrt{5^2 + (-1)^2}$
$= \sqrt{26}$
$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[2 - (-3)]^2 + [-3 - (-2)]^2}$
$= \sqrt{5^2 + (-1)^2}$
$= \sqrt{26}$

The measures of $\overline{AD}$ and $\overline{BC}$ are equal. Therefore $\overline{AD} \cong \overline{BC}$. Since $\overline{AD} \parallel \overline{BC}$ and $\overline{AD} \cong \overline{BC}$, quadrilateral $ABCD$ is a parallelogram; yes.

24. Let the vertices of the quadrilateral be $A(4, 11)$, $B(8, 14)$, $C(4, 19)$, and $D(0, 15)$.



A quadrilateral is a parallelogram if both pairs of opposite sides are parallel. $\overline{AB}$ and $\overline{DC}$ are one pair of opposite sides.

slope of $\overline{AB}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{14 - 11}{8 - 4}$
$= \frac{3}{2}$

slope of $\overline{DC}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{19 - 15}{4 - 0}$
$= \frac{4}{4}$ or 1

Since $\overline{AB} \nparallel \overline{DC}$, quadrilateral $ABCD$ is not a parallelogram; no.

25. The slope of the line through (15, 1) and (−3, −8) should be equal to the slope of the line through (−3, −8) and (3, k) since all three points lie on the same line.

slope through (15, 1) and (−3, −8)

$m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-8 - 1}{-3 - 15}$
$= \frac{-9}{-18}$ or $\frac{1}{2}$

slope through (−3, −8) and (3, k)

$m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{k - (-8)}{3 - (-3)}$
$= \frac{k + 8}{6}$

$\frac{k + 8}{6} = \frac{1}{2} \Rightarrow k = -5$

26. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{[-1 - (-3)]^2 + (2\sqrt{3} - 0)^2}$
$= \sqrt{16}$ or 4

$BC = \sqrt{[1 - (-1)]^2 + (0 - 2\sqrt{3})^2}$
$= \sqrt{16}$ or 4

$CA = \sqrt{(-3 - 1)^2 + (0 - 0)^2}$
$= \sqrt{16}$ or 4

Yes, $AB = 4$, $BC = 4$, and $CA = 4$. Thus, $\overline{AB} \cong \overline{BC} \cong CA$. Therefore the points A, B, and C form an equilateral triangle.

27. $EF = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(4 - 2)^2 + (4 - 5)^2}$
$= \sqrt{5}$

$HG = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(2 - 0)^2 + (0 - 1)^2}$
$= \sqrt{5}$

slope of $\overline{EF}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{4 - 5}{4 - 2}$ or $-\frac{1}{2}$

slope of $\overline{FG}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{0 - 4}{2 - 4}$ or $\frac{2}{1}$

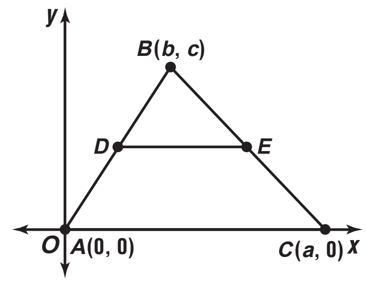
slope of $\overline{HG}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{0 - 1}{2 - 0}$ or $-\frac{1}{2}$

$\overline{EF} \cong \overline{HG}$ since $EF = \sqrt{5}$ and $HG = \sqrt{5}$.

$\overline{EF} \parallel \overline{HG}$ since the slope of $\overline{EF}$ is $-\frac{1}{2}$ and the slope of $\overline{HG}$ is $-\frac{1}{2}$. Thus the points form a parallelogram. $\overline{EF} \perp \overline{FG}$ since the product of the slopes of $\overline{EF}$ and $\overline{FG}$, $-\frac{1}{2} \cdot \frac{2}{1}$, is −1. Therefore, the points form a rectangle.

28. Let $A(0, 0)$, $B(b, c)$, and $C(a, 0)$ be the vertices of a triangle. Let D be the midpoint of $\overline{AB}$ and E be the midpoint of $\overline{BC}$.



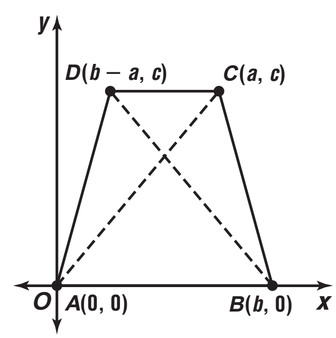
The coordinates of D are $\left(\frac{0 + b}{2}, \frac{0 + c}{2}\right)$ or $\left(\frac{b}{2}, \frac{c}{2}\right)$.
The coordinates of E are $\left(\frac{b + a}{2}, \frac{c + 0}{2}\right)$ or $\left(\frac{b + a}{2}, \frac{c}{2}\right)$.

$AC = \sqrt{(a - 0)^2 + (0 - 0)^2}$
$= \sqrt{a^2}$ or a

$DE = \sqrt{\left(\frac{b + a}{2} - \frac{b}{2}\right)^2 + \left(\frac{c}{2} - \frac{c}{2}\right)^2}$
$= \sqrt{\frac{a^2}{4}}$ or $\frac{a}{2}$

Since $DE = \frac{1}{2}AC$, the line segment joining the midpoints of two sides of a triangle is equal in length to one-half the third side.

29. In trapezoid $ABCD$, let A and B have coordinates (0, 0) and (b, 0), respectively. To make the trapezoid isosceles, let C have coordinates ($b - a$, c) and let D have coordinates (a, c).

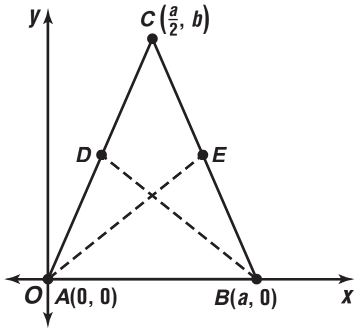


$AC = \sqrt{(a - 0)^2 + (c - 0)^2}$
$= \sqrt{a^2 + c^2}$

$BD = \sqrt{(b - a - b)^2 + (c - 0)^2}$
$= \sqrt{a^2 + c^2}$

$AC = \sqrt{a^2 + c^2} = \sqrt{a^2 + c^2} = BD$, so the diagonals of an isosceles trapezoid are congruent.

30. In $\triangle ABC$, let the vertices be $A(0, 0)$ and $B(a, 0)$. Since $\overline{AC}$ and $\overline{BC}$ are congruent sides, let the third vertex be $C\left(\frac{a}{2}, b\right)$. Let D be the midpoint of $\overline{AC}$ and let E be the midpoint of $\overline{BC}$.



The coordinates of D are: $\left(\frac{\frac{a}{2}+0}{2}, \frac{b+0}{2}\right)$ or $\left(\frac{a}{4}, \frac{b}{2}\right)$.

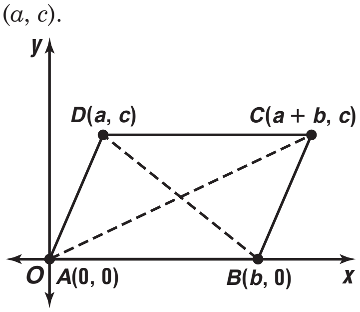
The coordinates of E are: $\left(\frac{\frac{a}{2}+a}{2}, \frac{b+0}{2}\right)$ or $\left(\frac{3a}{4}, \frac{b}{2}\right)$.

$AE = \sqrt{\left(\frac{3a}{4} - 0\right)^2 + \left(\frac{b}{2} - 0\right)^2} = \frac{1}{2}\sqrt{\frac{9a^2}{4} + b^2}$

$BD = \sqrt{\left(a - \frac{a}{4}\right)^2 + \left(0 - \frac{b}{2}\right)^2} = \frac{1}{2}\sqrt{\frac{9a^2}{4} + b^2}$

Since $AE = BD$, the medians to the congruent sides of an isosceles triangle are congruent.

31. Let A and B have coordinates $(0, 0)$ and $(b, 0)$ respectively. To make a parallelogram, let C have coordinates $(b + a, c)$ and let D have coordinates (a, c).

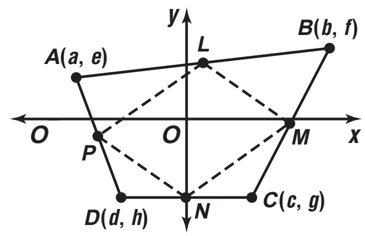


The midpoint of $\overline{BD}$ is $\left(\frac{a+b}{2}, \frac{0+c}{2}\right)$ or $\left(\frac{a+b}{2}, \frac{c}{2}\right)$.

The midpoint of $\overline{AC}$ is $\left(\frac{a+b+0}{2}, \frac{c+0}{2}\right)$ or $\left(\frac{a+b}{2}, \frac{c}{2}\right)$.

Since the diagonals have the same midpoint, the diagonals bisect each other.

32. Let the vertices of quadrilateral $ABCD$ be $A(a, e)$, $B(b, f)$, $C(c, g)$, and $D(d, h)$. The midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively, are $L\left(\frac{a+b}{2}, \frac{e+f}{2}\right)$, $M\left(\frac{b+c}{2}, \frac{f+g}{2}\right)$, $N\left(\frac{c+d}{2}, \frac{g+h}{2}\right)$, and $P\left(\frac{a+d}{2}, \frac{e+h}{2}\right)$.



The slope of $\overline{LM}$ is $\dfrac{\frac{f+g}{2} - \frac{e+f}{2}}{\frac{b+c}{2} - \frac{a+b}{2}}$ or $\frac{g-e}{c-a}$.

The slope of $\overline{NP}$ is $\dfrac{\frac{e+h}{2} - \frac{g+h}{2}}{\frac{a+d}{2} - \frac{c+d}{2}}$ or $\frac{e-g}{a-c}$.

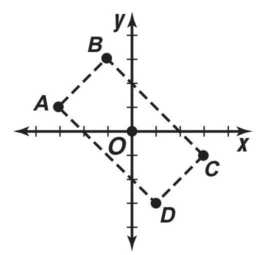
These slopes are equal, so $\overline{LM} \parallel \overline{NP}$.

The slope of $\overline{MN}$ is $\dfrac{\frac{f+g}{2} - \frac{g+h}{2}}{\frac{b+c}{2} - \frac{c+d}{2}}$ or $\frac{f-h}{b-d}$.

The slope of $\overline{PL}$ is $\dfrac{\frac{e+h}{2} - \frac{e+f}{2}}{\frac{a+d}{2} - \frac{a+b}{2}}$ or $\frac{h-f}{d-b}$.

These slopes are equal, so $\overline{MN} \parallel \overline{PL}$. Since $\overline{LM} \parallel \overline{NP}$, and $\overline{MN} \parallel \overline{PL}$, $PLMN$ is a parallelogram.

33. Let the vertices of the rectangle be $A(-3, 1)$, $B(1, 3)$, $C(3, -1)$, and $D(1, -3)$. Since the area of a rectangle is the length times the width, find the measure of two consecutive sides, $\overline{AD}$ and $\overline{DC}$.



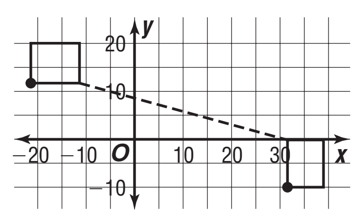
$$\begin{aligned} AD &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[1 - (-3)]^2 + (-3 - 1)^2} \\ &= \sqrt{4^2 + (-4)^2} \\ &= \sqrt{32} \text{ or } 4\sqrt{2} \end{aligned}$$

$$\begin{aligned} DC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - 1)^2 + [-1 - (-3)]^2} \\ &= \sqrt{2^2 + 2^2} \\ &= \sqrt{8} \text{ or } 2\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \ell w \\ &= (4\sqrt{2})(2\sqrt{2}) \\ &= 16 \end{aligned}$$

The area of the rectangle is 16 units^2.

34a.

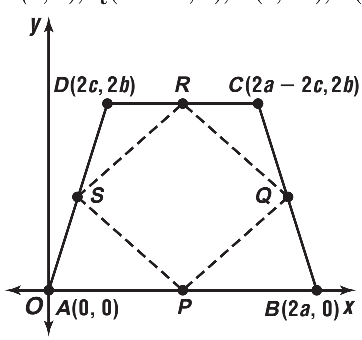


34b. The two regions are closest between $(-12, 12)$ and $(31,0)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[31 - (-12)]^2 + (0 - 12)^2}$
$= \sqrt{43^2 + (-12)^2}$
$= \sqrt{1993}$ or about 44,64

The distance between these two points is about 44.64 pixels, which is greater than 40 pixels. therefore, the regions meet the criteria.

35. Let the vertices of the isosceles trapezoid have the coordinates $A(0, 0)$, $B(2a, 0)$, $C(2a - 2c, 2b)$, $D(2c, 2b)$. The coordinates of the midpoints are: $P(a, 0)$, $Q(2a - c, b)$, $R(a, 2b)$, $S(c, b)$.



$PQ = \sqrt{(2a - c - a)^2 + (b - 0)^2}$
$= \sqrt{(a - c)^2 + b^2}$
$QR = \sqrt{(2a - c - a)^2 + (b - 2b)^2}$
$= \sqrt{(a - c)^3 + b^2}$
$RS = \sqrt{(a - c)^2 + (2b - b)^2}$
$= \sqrt{(a - c)^2 + b^2}$
$PS = \sqrt{(a - c)^2 + (0 - b)^2}$
$= \sqrt{(a - c)^2 + b^2}$

So, all of the sides are congruent and quadrilateral $PQRS$ is a rhombus.

36a. distance from fountain to rosebushes:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{[1 - (-3)]^2 + (-3 - 2)^2}$
$d = \sqrt{41}$ or $2\sqrt{41}$ meters

distance from rosebushes to bench:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(3 - 1)^2 + [3 - (-3)]^2}$
$d = 2\sqrt{10}$ or $4\sqrt{10}$ meters

distance from bench to fountain:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(-3 - 3)^2 + (2 - 3)^2}$
$d = \sqrt{37}$ or $2\sqrt{37}$ meters

Yes; the distance from the fountain to the rosebushes is $2\sqrt{41}$ or about 12.81 meters. The distance from the rosebushes to the bench is $4\sqrt{10}$ or about 12.65 meters. The distance from the bench to the fountain is $2\sqrt{37}$ or about 12.17 meters.

36b. The fountain is located at $(-3, 2)$ and the rosebushes are located at $(1, -3)$.

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3 + 1}{2}, \frac{2 + (-3)}{2}\right)$
$= \left(-1, -\frac{1}{2}\right)$

37a. Find a representation for MA and for MB.

$MA = \sqrt{t^2 + (3t - 15)^2}$
$= \sqrt{t^2 + 9t^2 - 90t + 225}$
$= \sqrt{10t^2 - 90t + 225}$
$MB = \sqrt{(t - 9)^2 + (3t - 12)^2}$
$= \sqrt{t^2 - 18t + 81 + 9t^2 - 72t + 144}$
$= \sqrt{10t^2 - 90t + 225}$

By setting these representations equal to each other, you find a value for t that would make the two distances equal.

$MA = MB$
$\sqrt{10t^2 - 90t + 225} = \sqrt{10t^2 - 90t + 225}$

Since the above equation is a true statement, t can take on any real values.

37b. A line; this line is the perpendicular bisector of $\overline{AB}$.

38. $r = \sqrt{a^2 + b^2}$ $\qquad \theta = \text{Arctan } \frac{b}{a}$
$= \sqrt{(-5)^2 + 12^2}$ $\qquad = \text{Arctan } \frac{12}{-5}$
$= \sqrt{169}$ or 13 $\qquad \approx -1.176005207$
$(-5 + 12\boldsymbol{i})^2 = 13^2 [\cos 2\theta + \boldsymbol{i} \sin 2\theta]$
$= -119 - 120\boldsymbol{i}$

39. If $\vec{\mathbf{v}} = (115, 2018, 0)$, then
$|\vec{\mathbf{v}}| = \sqrt{115^2 + 2018^2 + 0^2}$
$= \sqrt{4085549}$ or about 2021

The magnitude of the force is about 2021 N.

40. $2 \sec^2 x \stackrel{?}{=} \frac{1}{1 + \sin x} + \frac{1}{1 - \sin x}$
$2 \sec^2 x \stackrel{?}{=} \frac{(1 - \sin x) + (1 + \sin x)}{(1 + \sin x)(1 - \sin x)}$
$2 \sec^2 x \stackrel{?}{=} \frac{2}{1 - \sin^2 x}$
$2 \sec^2 x \stackrel{?}{=} = \frac{2}{\cos^2 x}$
$2 \sec^2 x = 2 \sec^2 x$

41. $s = r\theta$
$11.5 = 12\theta$
$\theta = \frac{11.5}{12}$ radians
$\frac{11.5}{12} \cdot \frac{180°}{\pi} \approx 54.9°$

42. $\sin 390° = \sin (390° - 360°)$
$= \sin 30°$ or $\frac{1}{2}$

43. $z^2 - 8z = -14$
$z^2 - 8z + 16 = -14 + 16$
$(z - 4)^2 = 2$
$z - 4 = \pm\sqrt{2}$
$z = 4 \pm \sqrt{2}$

44. $x^2 = 16$
$x = \pm\sqrt{16}$ or ± 4
$y^2 = 4$
$y = \pm\sqrt{4}$ or ± 2

Evaluating $(x - y)^2$ when $x = 4$ and $y = -2$ results in the greatest possible value, $[4 - (-2)]^2$ or 36.

10-2 Circles

Page 627 Check for Understanding

1. complete the square on each variable

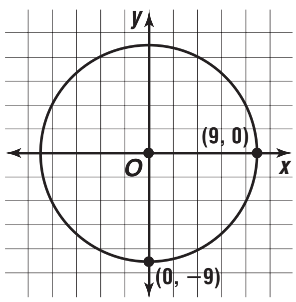
2. Sample answer: $(x + 4)^2 + (y - 9)^2 = 1$,
$(x + 4)^2 + (y - 9)^2 = 2$, $(x + 4)^2 + (y - 9)^2 = 3$,
$(x + 4)^2 + (y - 9)^2 = 4$, $(x + 4)^2 + (y - 9)^2 = 5$

3. Find the center of the circle, (h, k), by finding the midpoint of the diameter. Next find the radius of the circle, r, by finding the distance from the center to one endpoint. Then write the equation of the circle in standard form as $(x - h)^2 + (y - k)^2 = r^2$.

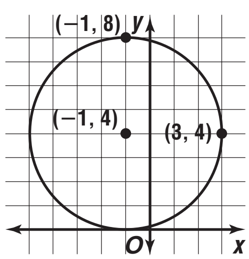
4. The equation $x^2 + y^2 + 8x + 8y + 36 = 0$ written in standard form is $(x + 4)^2 + (y + 4)^2 = -4$. Since a circle cannot have a negative radius, the graph of the equation is the empty set.

5. Ramon; the square root of a sum does not equal the sum of the square roots.

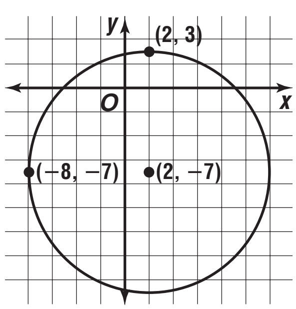
6.
$$\begin{aligned} (x - h)^2 + (y - k)^2 &= r^2 \\ (x - 0)^2 + (y - 0)^2 &= 9^2 \\ x^2 + y^2 &= 81 \end{aligned}$$



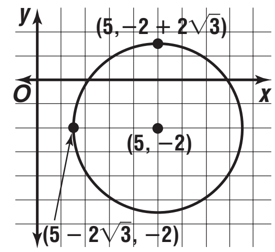
7.
$$\begin{aligned} (x - h)^2 + (y - k)^2 &= r^2 \\ [x - (-1)]^2 + (y - 4)^2 &= [3 - (-1)]^2 \\ (x + 1)^2 + (y - 4)^2 &= 16 \end{aligned}$$



8.
$$\begin{aligned} x^2 + y^2 - 4x + 14y - 47 &= 0 \\ x^2 - 4x + 4 + y^2 + 14y + 49 &= 47 + 4 + 49 \\ (x - 2)^2 + (y + 7)^2 &= 100 \end{aligned}$$



9.
$$\begin{aligned} 2x^2 + 2y^2 - 20x + 8y + 34 &= 0 \\ 2x^2 - 20x + 2y^2 + 8y &= -34 \\ 2(x^2 - 10x + 25) + 2(y^3 + 4y + 4) &= -34 + 2(25) \\ &\quad + 2(4) \\ 2(x - 5)^2 + 2(y + 2)^2 &= 24 \\ (x - 5)^2 + (y + 2)^2 &= 12 \end{aligned}$$



10.
$$\begin{aligned} x^2 + y^2 + Dx + Ey + F &= 0 \\ 0^2 + 0^2 + D(0) + E(0) + F &= 0 \\ 4^2 + 0^2 + D(4) + E(0) + F &= 0 \\ 0^2 + 4^2 + D(0) + E(4) + F &= 0 \end{aligned}$$

$F = 0$ $\qquad F = 0$
$4D + F = -16 \Rightarrow D = -4$
$4E + F = -16 \qquad E = -4$

$$\begin{aligned} x^2 + y^2 - 4x - 4y &= 0 \\ x^2 - 4x + 4 + y^2 - 4y + 4 &= 0 + 4 + 4 \\ (x - 2)^2 + (y - 2)^2 &= 8 \end{aligned}$$

center: $(h, k) = (2, 2)$
radius: $r^2 = 8$
$r = \sqrt{8}$ or $2\sqrt{2}$

11.
$$\begin{aligned} x^2 + y^2 + Dx + Ey + F &= 0 \\ 1^2 + 3^2 + D(1) + E(3) + F = 0 &\Rightarrow D + 3E + F = -10 \\ 5^2 + 5^2 + D(5) + E(5) + F = 0 &\Rightarrow 5D + 5E + F = -50 \\ 5^2 + 3^2 + D(5) + E(3) + F = 0 &\Rightarrow 5D + 3E + F = -34 \end{aligned}$$

$$\begin{aligned} D + 3E + F &= -10 \\ \underline{(-1)(5D + 5E + F)} &\underline{= (-1)(-50)} \\ -4D - 2E \quad &= 40 \end{aligned}$$

$$\begin{aligned} 5D + 5E + F &= -50 \\ \underline{(-1)(5D + 3E + F)} &\underline{= (-1)(-34)} \\ 2E \quad &= -16 \\ E &= -8 \end{aligned}$$

$$\begin{aligned} -4D - 2(-8) &= 40 \\ -4D &= 24 \\ D &= -6 \end{aligned} \qquad \begin{aligned} (-6) + 3(-8) + F &= -10 \\ F &= 20 \end{aligned}$$

$$\begin{aligned} x^2 + y^2 - 6x - 8y + 20 &= 0 \\ x^2 - 6x + 9 + y^2 - 8y + 16 &= -20 + 9 + 16 \\ (x - 3)^2 + (y - 4)^2 &= 5 \end{aligned}$$

center: $(h, k) = (3, 4)$
radius: $r^2 = 5$
$r = \sqrt{5}$

12.
$$\begin{aligned} (x - h)^2 + (y - k)^2 &= r^2 \\ [x - (-2)]^2 + (y - 1)^2 &= r^2 \\ (x + 2)^2 + (y - 1)^2 &= r^2 \\ (1 + 2)^2 + (5 - 1)^2 &= r^2 \\ 25 &= r^2 \\ (x + 2)^2 + (y - 1)^2 &= 25 \end{aligned}$$

13. midpoint of diameter:

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+10}{2}, \frac{6+(-10)}{2}\right)$$
$$= (4, -2)$$

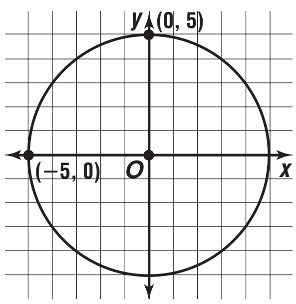
radius: $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$$= \sqrt{[4-(-2)]^2 + [6-(-2)]^2}$$
$$= \sqrt{6^2 + 8^2}$$
$$= \sqrt{100} \text{ or } 10$$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-4)^2 + [y-(-2)]^2 = 10^2$$
$$(x-4)^2 + (y+2)^2 = 100$$

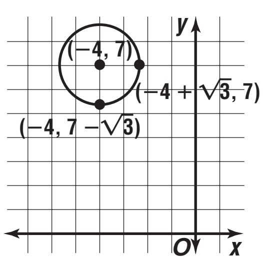
14. $(x-h)^2 + (y-k)^2 = r^2$
$$(x-0)^2 + (y-0)^2 = (1740 + 185)^2$$
$$x^2 + y^2 = 1925^2$$

Pages 627–630 Exercises

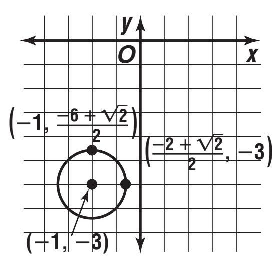
15. $(x-h)^2 + (y-k)^2 = r^2$
$$(x-0)^2 + (y-0)^2 = 5^2$$
$$x^2 + y^2 = 25$$



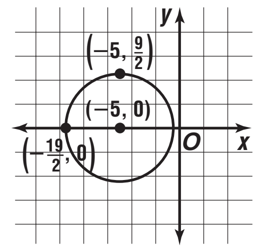
16. $(x-h)^2 + (y-k)^2 = r^2$
$$[x-(-4)]^2 + (y-7)^2 = (\sqrt{3})^2$$
$$(x+4)^2 + (y-7)^2 = 3$$



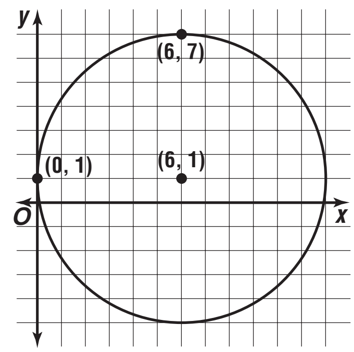
17. $(x-h)^2 + (y-k)^2 = r^2$
$$[x-(-1)]^2 + [y-(-3)]^2 = \left(\frac{\sqrt{2}}{2}\right)^2$$
$$(x+1)^2 + (y+3)^2 = \frac{1}{2}$$



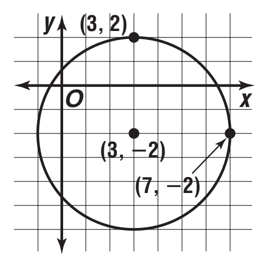
18. $(x-h)^2 + (y-k)^2 = r^2$
$$[x-(-5)]^2 + (y-0)^2 = \left(\frac{9}{2}\right)^2$$
$$(x+5)^2 + y^2 = \frac{81}{4}$$



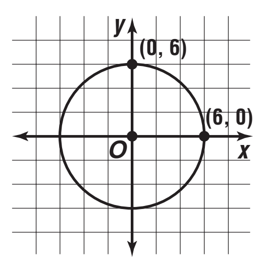
19. $(x-h)^2 + (y-k)^2 = r^2$
$$(x-6)^2 + (y-1)^2 = 6^2$$
$$(x-6)^2 + (y-1)^2 = 36$$



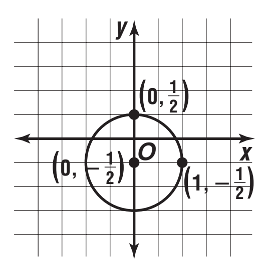
20. $(x-h)^2 + (y-k)^2 = r^2$
$$(x-3)^2 + [y-(-2)]^2 = [2-(-2)]^2$$
$$(x-3)^2 + (y+2)^2 = 16$$



21. $36 - x^2 = y^2$
$$x^2 + y^2 = 36$$

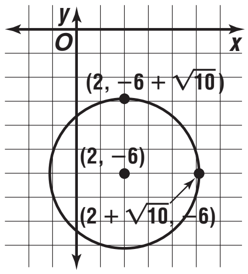


22. $x^2 + y^2 + y = \frac{3}{4}$
$$x^2 + y^2 + y + \frac{1}{4} = \frac{3}{4} + \frac{1}{4}$$
$$x^2 + \left(y + \frac{1}{2}\right)^2 = 1$$



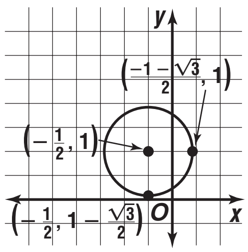
23.

$$\begin{aligned} x^2 + y^2 - 4x + 12y + 30 &= 0 \\ x^2 - 4x + 4 + y^2 + 12y + 36 &= -30 + 4 + 36 \\ (x - 2)^2 + (y + 6)^2 &= 10 \end{aligned}$$



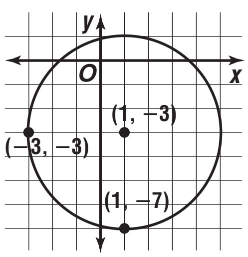
24.

$$\begin{aligned} 2x^2 + 2y^2 + 2x - 4y &= -1 \\ 2x^2 + 2x + 2y^2 - 4y &= -1 \\ 2\left(x^2 + 1x + \tfrac{1}{4}\right) + 2(y^2 - 2y + 1) &= -1 + 2\left(\tfrac{1}{4}\right) + 2(1) \\ 2\left(x + \tfrac{1}{2}\right)^2 + 2(y - 1)^2 &= \tfrac{3}{2} \\ \left(x + \tfrac{1}{2}\right)^2 + (y - 1)^2 &= \tfrac{3}{4} \end{aligned}$$



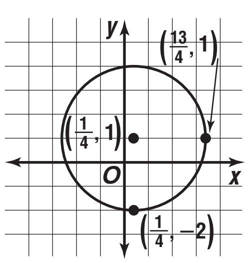
25.

$$\begin{aligned} 6x^2 - 12x + 6y^2 + 36y &= 36 \\ 6(x^2 - 2x + 1) + 6(y^2 + 6y + 9) &= 36 + 6(1) + 6(9) \\ 6(x - 1)^2 + 6(y + 3)^2 &= 96 \\ (x - 1)^2 + (y + 3)^2 &= 16 \end{aligned}$$



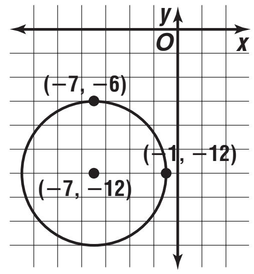
26.

$$\begin{aligned} 16x^2 + 16y^2 - 8x - 32y &= 127 \\ 16x^2 - 8x + 16y^2 - 32y &= 127 \\ 16\left(x^2 - \tfrac{1}{2}x + \tfrac{1}{16}\right) + 16(y^2 - 2y + 1)^2 &= 127 + 16\left(\tfrac{1}{16}\right) \\ &\quad + 16(1) \\ 16\left(x - \tfrac{1}{4}\right)^2 + 16(y - 1)^2 &= 144 \\ \left(x - \tfrac{1}{4}\right)^2 + (y - 1)^2 &= 9 \end{aligned}$$



27.

$$\begin{aligned} x^2 + y^2 + 14x + 24y + 157 &= 0 \\ x^2 + 14x + 49 + y^2 + 24y + 144 &= -157 + 49 + 144 \\ (x + 7)^2 + (y + 12)^2 &= 36 \end{aligned}$$



28.

$$\begin{aligned} x^2 + y^2 + Dx + Ey + F &= 0 \\ 0^2 + (-1)^2 + D(0) + E(-1) + F &= 0 \Rightarrow \\ -E + F &= -1 \\ (-3)^2 + (-2)^2 + D(-3) + E(-2) + F &= 0 \Rightarrow \\ -3D - 2E + F &= -13 \\ (-6)^2 + (-1)^2 + D(-6) + E(-1) + F &= 0 \Rightarrow \\ -6D - E + F &= -37 \end{aligned}$$

$$\begin{aligned} -\ E + F &= -1 \\ (-1)(-3D - 2E + F) &= (-1)(-13) \\ \hline 3D + E &= 12 \end{aligned}$$

$$\begin{aligned} -3D - 2E + F &= -13 \\ (-1)(-6D - E + F) &= (-1)(-37) \\ \hline 3D - E &= 24 \end{aligned}$$

$$\begin{aligned} 3D + E &= 12 \qquad & 3(6) + E &= 12 \\ 3D - E &= 24 & E &= -6 \\ \hline 6D &= 36 \\ D &= 6 & -(-6) + F &= -1 \\ & & F &= -7 \end{aligned}$$

$$\begin{aligned} x^2 + y^2 + 6x - 6y - 7 &= 0 \\ x^2 + 6x + 9 + y^2 - 6y + 9 &= 7 + 9 + 9 \\ (x + 3)^2 + (y - 3)^2 &= 25 \end{aligned}$$

center: $(h, k) = (-3, 3)$

radius: $r^2 = 25$

$r = \sqrt{25}$ or 5

29.

$$\begin{aligned} x^2 + y^2 + Dx + Ey + F &= 0 \\ 7^2 + (-1)^2 + D(7) + E(-1) + F &= 0 \Rightarrow \\ 7D - E + F &= -50 \\ 11^2 + (-5)^2 + D(11) + E(-5) + F &= 0 \Rightarrow \\ 11D - 5E + F &= -146 \\ 3^2 + (-5)^2 + D(3) + E(-5) + F &= 0 \Rightarrow \\ 3D - 5E + F &= -34 \end{aligned}$$

$$\begin{aligned} 7D - E + F &= -50 \\ (-1)(11D - 5E + F) &= -1(-146) \\ \hline -4D + 4E &= 96 \end{aligned}$$

$$\begin{aligned} 11D - 5E + F &= -146 \\ (-1)(3D - 5E + F) &= -1(-34) \\ \hline 8D &= -12 \\ D &= -14 \end{aligned}$$

$$\begin{aligned} -4(-14) + 4E &= 96 \\ 4E &= 40 \qquad & 7(-14) - (10) + F &= -50 \\ E &= 10 & F &= 58 \end{aligned}$$

$$\begin{aligned} x^2 + y^2 - 14x + 10y + 58 &= 0 \\ x^2 - 14x + 49 + y^2 + 10y + 25 &= -58 + 49 + 25 \\ (x - 7)^2 + (y + 5)^2 &= 16 \end{aligned}$$

center: $(h, k) = (7, -5)$

radius: $r^2 = 16$

$r = \sqrt{16}$ or 4

30. $x^2 + y^2 + Dx + Ey + F = 0$

$(-2)^2 + 7^2 + D(-2) + E(7) + F = 0 \Rightarrow$
$-2D + 7E + F = -53$

$(-9)^2 + 0^2 + D(-9) + E(0) + F = 0 \Rightarrow$
$-9D + F = -81$

$(-10)^2 + (-5)^2 + D(-10) + E(-5) + F = 10 \Rightarrow$
$-10D - 5E + F = -125$

$$\begin{array}{l} -2D + 7E + F = -53 \\ \underline{(-1)(-9D \qquad + F) = (-1)(-81)} \\ 7D + 7E \qquad = 28 \\ D + E = 4 \end{array}$$

$$\begin{array}{l} -10D - 5E + F = -125 \\ \underline{(-1)(-9D \qquad + F) = (-1)(-81)} \\ -D - 5E \qquad = -44 \end{array}$$

$$\begin{array}{l} -D - 5E = -44 \\ \underline{D + \ \ E = 4} \\ -4E = -40 \\ E = 10 \end{array}$$

$D + (10) = 4$
$D = -6$
$-9(-6) + F = -81$
$F = -135$

$x^2 + y^2 - 6x + 10y - 135 = 10$
$x^2 - 6x + 9 + y^2 + 10y + 25 = 135 + 9 + 25$
$(x - 3)^2 + (y + 5)^2 = 169$
center: $(h, k) = (3, -5)$
radius: $r^2 = 169$
$r = \sqrt{169}$ or 13

31. $x^2 + y^2 + Dx + Ey + F = 0$

$(-2)^2 + 3^2 + D(-2) + E(3) + F = 0 \Rightarrow$
$-2D + 3E + F = -13$

$6^2 + (-5)^2 + D(6) + E(-5) + F = 0 \Rightarrow$
$6D - 5E + F = -61$

$0^2 + 7^2 + D(0) + E(7) + F = 0 \Rightarrow$
$7E + F = -49$

$$\begin{array}{l} -2D + 3E + F = -13 \\ \underline{(-1)(6D - 5E + F) = (-1)(-61)} \\ -8D + 8E \qquad = 48 \\ -D + E = 6 \end{array}$$

$$\begin{array}{l} 6D - \ 5E + F = -61 \\ \underline{(-1)(7E + F) = (-1)(-49)} \\ 6D - 12E \qquad = -12 \\ D - 2E = -2 \end{array}$$

$$\begin{array}{l} -D + \ \ E = 6 \\ \underline{D - 2E = -2} \\ -E = 4 \\ E = -4 \end{array}$$

$D - 2(-4) = -2$
$D = -10$
$7(-4) + F = -49$
$F = -21$

$x^2 + y^2 - 10x - 4y - 21 = 0$
$x^2 - 10x + 25 + y^2 - 4y + 4 = 21 + 25 + 4$
$(x - 5)^2 + (y - 2)^2 = 50$
center: $(h, k) = (5, 2)$
radius: $r^2 = 50$
$r = \sqrt{50}$ or $5\sqrt{2}$

32. $x^2 + y^2 + Dx + Ey + F = 0$

$4^2 + 5^2 + D(4) + E(5) + F = 0 \Rightarrow$
$4D + 5E + F = -41$

$(-2)^2 + 3^2 + D(-2) + E(3) + F = 0 \Rightarrow$
$-2D + 3E + F = -13$

$(-4)^2 + (-3)^2 + D(-4) + E(-3) + F = 0 \Rightarrow$
$-4D - 3E + F = -25$

$$\begin{array}{l} 4D + 5E + F = -41 \\ \underline{(-1)(-2D + 3E + F) = (-1)(-13)} \\ 6D + 2E \qquad = -28 \\ 3D + E = -14 \end{array}$$

$$\begin{array}{l} 4D + 5E + F = -41 \\ \underline{(-1)(-4D - 3E + F) = (-1)(-25)} \\ 8D + 8E \qquad = -16 \\ D + E = -2 \end{array}$$

$$\begin{array}{l} 3D + E = -14 \\ \underline{(-1)(D + E) = (-1)(-2)} \\ 2D \qquad = -12 \\ D = -6 \end{array}$$

$-6 + E = -2$
$E = 4$
$-2(-6) + 3(4) + F = -13$
$F = -37$

$x^2 + y^2 - 6x + 4y - 37 = 0$
$x^2 + 6x + 9 + y^2 + 4y + 4 = 37 + 9 + 4$
$(x - 3)^2 + (y + 2)^2 = 50$
center: $(h, k) = (3, -2)$
radius: $r^2 = 50$
$r = \sqrt{50}$ or $5\sqrt{2}$

33. $x^2 + y^2 + Dx + Ey + F = 0$

$1^2 + 4^2 + D(1) + E(4) + F = 0 \Rightarrow$
$D + 4E + F = -17$

$2^2 + (-1)^2 + D(2) + E(-1) + F = 0 \Rightarrow$
$2D - E + F = -5$

$(-3)^2 + 0^2 + D(-3) + E(0) + F = 0 \Rightarrow$
$-3D + F = -9$

$$\begin{array}{l} D + 4E + F = -17 \\ \underline{(-1)(2D - \ \ E + F) = (-1)(-5)} \\ -D + 5E \qquad = -12 \end{array}$$

$$\begin{array}{l} D + 4E + F = -17 \\ \underline{(-1)(-3D + \qquad F) = (-1)(-9)} \\ 4D + 4E \qquad = -8 \\ D + E = -2 \end{array}$$

$$\begin{array}{l} -D + 5E = -12 \\ \underline{D + \ \ E = -2} \\ 6E = -14 \\ E = -\frac{7}{3} \end{array}$$

$D + \left(-\frac{7}{3}\right) = -2$
$D = \frac{1}{3}$
$-3\left(\frac{1}{3}\right) + F = -9$
$F = -8$

$x^2 + y^2 + Dx + Ey + F = 0$
$x^2 + y^2 + \frac{1}{3}x - \frac{7}{3}y - 8 = 0$
$x^2 + \frac{1}{3}x + \frac{1}{36} + y^2 - \frac{7}{3}y + \frac{49}{36} = 8 + \frac{1}{36} + \frac{49}{36}$
$\left(x + \frac{1}{6}\right)^2 + \left(y - \frac{7}{6}\right)^2 = \frac{169}{18}$

center: $(h, k) = \left(-\frac{1}{6}, \frac{7}{6}\right)$

radius: $r^2 = \frac{169}{18}$

$r = \sqrt{\frac{169}{18}}$

$= \frac{13}{3\sqrt{2}}$ or $\frac{13\sqrt{2}}{6}$

34. $x^2 + y^2 + Dx + Ey + F = 0$

$0^2 + 0^2 + D(0) + E(0) + F = 0 \Rightarrow$

$F = 0$

$(2.8)^2 + 0^2 + D(2.8) + E(0) + F = 0 \Rightarrow$

$2.8D + F = -7.84$

$(5)^2 + 2^2 + D(5) + E(2) + F = 0 \Rightarrow$

$5D + 2E + F = -29$

$2.8D + 0 = -7.84$ $\quad$ $5(-2.8) + 2E + (0) = -29$

$2.8D = -7.84$ $\quad$ $2E = -15$

$D = -2.8$ $\quad$ $E = -7.5$

$x^2 + y^2 - 2.8x - 7.5y + 0 = 0$

$x^2 - 2.8x + 1.96 + y^2 - 7.5y + 14.0625 = 1.96 + 14.0625$

$(x - 1.4)^2 + (y - 3.75)^2 = 16.0225$

or about $(x - 1.4)^2 + (y - 3.75)^2 = 16.02$

35. $(x - h)^2 + (y - k)^2 = r^2$

$[x - (-4)]^2 + (y - 3)^2 = r^2$

$(x + 4)^2 + (y - 3)^2 = r^2$

$(0 + 4)^2 + (0 - 3)^2 = r^2$

$25 = r^2$

$(x + 4)^2 + (y - 3)^2 = 25$

36. $(x - h)^2 + (y - k)^2 = r^2$

$(x - 2)^2 + (y - 3)^2 = r^2$

$(5 - 2)^2 + (6 - 3)^2 = r^2$

$18 = r^2$

$(x - 2)^2 + (y - 3)^2 = 18$

37. midpoint of diameter:

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + (-6)}{2}, \frac{3 + (-5)}{2}\right)$

$= (-2, -1)$

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-2 - 2)^2 + (-1 - 3)^2}$

$= \sqrt{(-4)^2 + (-4)^2}$

$= \sqrt{32}$

$(x - h)^2 + y - k)^2 = r^2$

$[x - (-2)]^2 + [y - (-1)]^2 = \left(\sqrt{32}\right)^2$

$(x + 2)^2 + (y + 1)^2 = 32$

38. midpoint of diameter:

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3 + 2}{2}, \frac{4 + 1}{2}\right)$

$= \left(-\frac{1}{2}, \frac{5}{2}\right)$

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left(-\frac{1}{2} - 2\right)^2 + \left(\frac{5}{2} - 1\right)^2}$

$= \sqrt{\left(-\frac{5}{2}\right)^2 + \left(\frac{3}{2}\right)^2}$

$= \sqrt{\frac{17}{2}}$

$(x - h)^2 + (y - k)^2 = r^2$

$\left[x - \left(-\frac{1}{2}\right)\right]^2 + \left(y - \frac{5}{2}\right)^2 = \left(\sqrt{\frac{17}{2}}\right)^2$

$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{17}{2}$

39. $(x - h)^2 + (y - k)^2 = r^2$

$(x - 5)^2 + (y - 1)^2 = r^2$

$x + 3y = -2$

$x + 3y + 2 = 0 \Rightarrow A = 1, B = 3,$ and $C = 2$

$r = \left|\frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}\right|$

$= \left|\frac{(1)(5) + (3)(1) + 2}{-\sqrt{1^2 + 3^2}}\right|$

$= \left|\frac{10}{-\sqrt{10}}\right|$ or $\sqrt{10}$

$(x - 5)^2 + (y - 1)^2 = \left(\sqrt{10}\right)^2$

$(x - 5)^2 + (y - 1)^2 = 10$

40. center: $(h, 0)$, radius: $r = 1$

$(x - h)^2 + (y - k)^2 = r^2$

$\left(\frac{\sqrt{2}}{2} - h\right)^2 + \left(\frac{\sqrt{2}}{2} - 0\right)^2 = 1^2$

$-\frac{1}{2} - \sqrt{2}h + h^2 + \frac{1}{2} = 1$

$h^2 - \sqrt{2}h = 1 - 1$

$h\left(h - \sqrt{2}\right) = 0$

$h = 0$ or $h = \sqrt{2}$

$(x - 0)^2 + (y - 0)^2 = 1$ $\quad$ $\left(x - \sqrt{2}\right)^2 + (y - 0)^2 = 1$

$x^2 + y^2 = 1$ $\quad$ or $\quad$ $\left(x - \sqrt{2}\right)^2 + y^2 = 1$

41a. $(x - h)^2 + (y - k)^2 = r^2$

$(x - 0)^2 + (y - 0)^2 = \left(\frac{12}{2}\right)^2$

$x^2 + y^2 = 36$

41b. $x^2 + y^2 = 36$

$y^2 = 36 - x^2$

$y = \pm\sqrt{36 - x^2}$

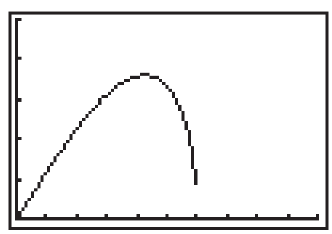
dimensions of rectangle:

$2x$ by $2y \Rightarrow 2x$ by $2\sqrt{36 - x^2}$

41c. $A(x) = 2x\left(2\sqrt{36 - x^2}\right)$

$= 4x\sqrt{36 - x^2}$

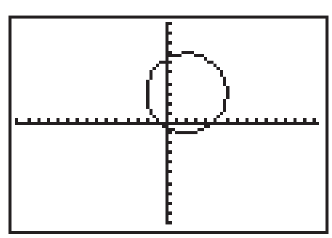
41d.



[0, 10] scl:1 by [0, 100] scl:20

41e. Use 4: maximum on the CALC menu of the calculator. The x-coordinate of this point is about 4.2. The maximum area of the rectangle is the corresponding y-value of 72, for an area of 72 units^2.

42a.

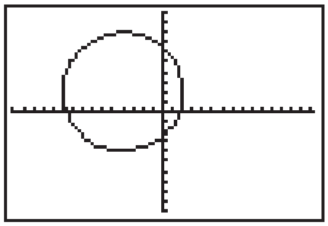


[−15.16, 15.16] scl:1 by [−5, 5] scl:1

42b. a circle centered at (2, 3) with radius 4

42c. $(x - 2)^2 + (y - 3)^2 = 16$

42d. center: $(h, k) = (-4, 2)$
radius: $r^2 = 36$
$r = \sqrt{36}$ or 6
$\boxed{\text{2nd}}$ [DRAW] 9:Circle($\boxed{-}$ 4 $\boxed{,}$ 2 $\boxed{,}$ 6 $\boxed{)}$



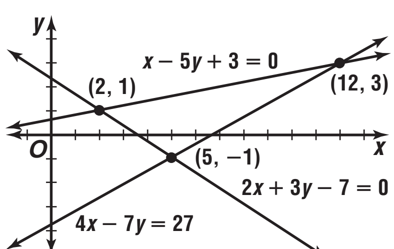
$[-15.16, 15.16]$ scl:1 by $[5, 5]$ scl:1

43a. $(x - h)^2 + (y - k)^2 = r^2$
$(x - 0)^2 + (y - 0)^2 = \left(\frac{24}{2}\right)^2$
$x^2 + y^2 = 144$

43b. $x^2 + y^2 = 6.25 \Rightarrow r_1{}^2 = 6.25$
$r_1 = \sqrt{6.25}$ or 2.5
If the circles are equally spaced apart then radius r_2 of the middle circle is found by adding the radius of the smallest circle to the radius of the largest circle and dividing by two.
$r_2 = \frac{12 + 2.5}{2}$ or 7.25

$$\begin{array}{c} \text{area of} \\ \text{region } B \end{array} = \begin{array}{c} \text{area of} \\ \text{middle circle} \end{array} - \begin{array}{c} \text{area of} \\ \text{smallest circle} \end{array}$$
$= \pi r_2{}^2 - \pi r_1{}^2$
$= \pi\,(r_2{}^2 - r_1{}^2)$
$= \pi\,(7.25^2 - 2.5^2)$
$= \pi\,(46.3125)$ or about 145.50
The area of region B is about 145.50 in^2.

44.



$x^2 + y^2 + Dx + Ey + F = 0$
$2^2 + 1^2 + D(2) + E(1) + F = 0 \Rightarrow$
$2D + E + F = -5$
$5^2 + (-1)^2 + D(5) + E(-1) + F = 0 \Rightarrow$
$5D - E + F = -26$
$12^2 + 3^2 + D(12) + E(3) + F = 0 \Rightarrow$
$12D + 3E + F = -153$

$$\begin{array}{l} 2D + E + F = -5 \\ \underline{(-1)(5D - E + F) = (-1)(-26)} \\ -3D + 2E \qquad = 21 \end{array}$$

$$\begin{array}{l} 2D + E + F = -5 \\ \underline{(-1)(12D + 3E + F) = (-1)(-153)} \\ -10D - 2E \qquad = 148 \end{array}$$

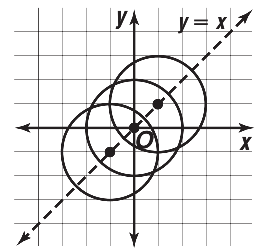
$$\begin{array}{l} -3D + 2E = 21 \\ \underline{-10D - 2E = 148} \\ -13D \qquad = 169 \end{array}$$

$D = -13$
$-3(-13) + 2E = 21$
$2E = -18$
$E = -9$

$2(-13) + (-9) + F = -5$
$F = 30$
$x^2 + y^2 - 13x - 9y + 30 = 0$
$x^2 - 13x + 42.25 + y^2 - 9y + 20.25 = -30 + 42.25 + 20.25$
$(x - 6.5)^2 + (y - 4.5)^2 = 32.5$

45a. $(x - h)^2 + (y - k)^2 = r^2$
$(x - k)^2 + (y - k)^2 = 2^2$
$(x - k)^2 + (y - k)^2 = 4$

45b. $(x - 1)^2 + (y - 1)^2 = 4$
$(x - 0)^2 + (y - 0)^2 = 4$
$(x + 1)^2 + (y + 1)^2 = 4$



45c. All of the circles in this family have a radius of 2 and centers located on the line $y = x$.

46a. $(x - h)^2 + (y - k)^2 = r^2$
$(x - 0)^2 + (y - 0)^2 = 14^2$
$x^2 + y^2 = 196$
$y^2 = 196 - x^2$
$y = \pm\sqrt{196 - x^2}$
$y = \sqrt{196 - x^2}$

46b. No, if $x = 7$, then $y = \sqrt{147} \approx 12.1$ ft, so the truck cannot pass.

47. $x^2 + y^2 - 8x + 6y + 25 = 0$
$x^2 - 8x + 16 + y^2 + 6y + 9 = -25 + 16 + 9$
$(x - 4)^2 + (y + 3)^2 = 0$
radius: $r^2 = 0$
$r = \sqrt{0}$ or 0
center: $(h, k) = (4, -3)$
Graph is a point located at $(4, -3)$.

48a. $(x - h)^2 + (y - k)^2 = r^2$
$(x - 0)^2 + (y - 0)^2 = r^2$
$x^2 + y^2 = r^2$
$(475)^2 + (1140)^2 = r^2$
$1{,}525{,}225 = r^2$
$x^2 + y^2 = 1{,}525{,}225$

48b. $r^2 = 1{,}525{,}225$
$r = \sqrt{1{,}525{,}225}$ or 1235
$A = \pi r^2$
$= \pi\,(1235)^2$ or approximately 4,792,000 ft^2

48c. $\frac{2500^2 - 4{,}792{,}000}{2500^2} = 0.23328$
about 23%

49a. $\overline{PA}$ has a slope of $\frac{y - 4}{x - 3}$ and $\overline{PB}$ has slope of $\frac{y + 4}{x + 3}$. If $\overline{PA} \perp \overline{PB}$ then $\frac{y - 4}{x - 3} \cdot \frac{y + 4}{x + 3} = -1$.
$\frac{y - 4}{x - 3} \cdot \frac{y + 4}{x + 3} = -1$
$\frac{y^2 - 16}{x^2 - 9} = -1$
$y^2 - 16 = -x^2 + 9$
$x^2 + y^2 = 25$

49b. If $\overline{PA} \perp \overline{PB}$, then A, P, and B are on the circle $x^2 + y^2 = 25$.

50. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(-2 - 4)^2 + [6 - (-3)]^2}$
$d = \sqrt{(-6)^2 + 9^2}$
$d = \sqrt{117}$

51. $(2 + \boldsymbol{i})(3 - 4\boldsymbol{i})(1 + 2\boldsymbol{i}) = (6 - 8\boldsymbol{i} + 3\boldsymbol{i} - 4\boldsymbol{i}^2)(1 + 2\boldsymbol{i})$
$= [6 - 8\boldsymbol{i} + 3\boldsymbol{i} - 4(-1)](1 + 2\boldsymbol{i})$
$= (10 - 5\boldsymbol{i})(1 + 2\boldsymbol{i})$
$= 10 + 20\boldsymbol{i} - 5\boldsymbol{i} - 10\boldsymbol{i}^2$
$= 10 + 20\boldsymbol{i} - 5\boldsymbol{i} - 10(-1)$
$= 20 + 15\boldsymbol{i}$

52. $x = t|\vec{\mathbf{v}}| \cos\theta$ $\quad y = t|\vec{\mathbf{v}}| \sin\theta - \frac{1}{2}gt^2$
$x = t(60)\cos 60°$ $\quad y = t(60)\sin 60° - \frac{1}{2}(32)t^2$
$x = 60t\cos 60°$ $\quad y = 60t\sin 60° - 16t^2$
$x = 60(0.5)\cos 60°$ $\quad y = 60(0.5)\sin 60° - 16(0.5)^2$
$x = 15$ $\quad y \approx 21.98076211$
15 ft horizontally, about 22 ft vertically

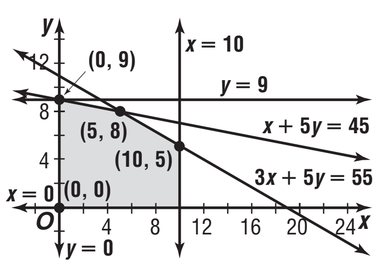
53. $A = \frac{5}{2}$ or 2.5
$20 = \frac{2\pi}{k}$ and $k = \frac{\pi}{10}$
$y = A\cos(kt)$
$y = 2.5\cos\left(\frac{\pi}{10}t\right)$

54. $s = \frac{1}{2}(a + b + c)$
$= \frac{1}{2}(15 + 25 + 35)$
$= 37.5$
$k = \sqrt{s(s - a)(s - b)(s - c)}$
$= \sqrt{37.5(37.5 - 15)(37.5 - 25)(37.5 - 3.5)}$
$= \sqrt{26{,}367.1875}$
≈ 162 units^2

55. $v = \sqrt{v_0^2 + 64h}$
$95 = \sqrt{15^2 + 64h}$
$95^2 = 15^2 + 64h$
$h = \frac{95^2 - 15^2}{64}$
$h = 137.5$ ft

56. $y = 6x^4 - 3x^2 + 1$
$b = 6a^4 - 3a^2 + 1$ $\quad (x, y) = (a, b)$
x-axis: $(x, y) = (a, -b)$
$-b = 6a^4 - 3a^2 + 1$; no
y-axis: $(x, y) = (-a, b)$
$b = 6(-a)^4 - 3(-a)^2 + 1$
$b = 6a^4 - 3a^2 + 1$; yes
$y = x$: $(x, y) = (b, a)$
$a = 6b^4 - 3b^2 + 1$; no
origin: $f(-x) = -f(x)$
$f(-x) = 6(-x)^4 - 3(-x)^2 + 1$ $\quad -f(x) = -(6x^4 - 3x^2 + 1)$
$f(-x) = 6x^4 - 3x^2 + 1$ $\quad -f(x) = -6x^4 + 3x^2 - 1$
no
The graph is symmetric with respect to the y-axis.

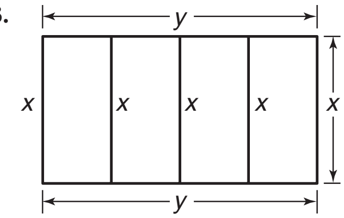
57a. Let x = number of cases of drug A.
Let y = number of cases of drug B.
$x = \leq 10$
$y \leq 9$
$3x + 5y \leq 55$
$x + 5y \leq 45$



(0, 0), (0, 9), (5, 8), (10, 5)
$f(x, y) = 320x + 500y$
$f(0, 0) = 320(0) + 500(0) = 0$
$f(0, 0) = 320(0) + 500(9) = 4500$
$f(5, 8) = 320(5) + 500(8) = 5600$
$f(10, 5) = 320(10) + 500(5) = 5700$
The maximum profit occurs when 10 cases of drug A and 5 cases of drug B are produced.

57b. When 10 cases of drug A and 5 cases of drug B are produced, the profit is $5700.

58.



$x + x + x + x + x + y + y = 5x + 2x$
The correct choice is A

10-3 Ellipses

Pages 637–638 Check For Understanding

1. $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
$\frac{y^2}{8^2} + \frac{x^2}{5^2} = 1$
$\frac{y^2}{64} + \frac{x^2}{25} = 1$

2. Since the foci lie on the major axis, determine whether the major axis is horizontal or vertical. If the a^2 is the denominator of the x terms, the major axis is horizontal. If the a^2 is the denominator of the y terms, the major axis is vertical.

3. When the foci and center of an ellipse coincide, $c = 0$.
$c^2 = a^2 - b^2$ $\quad e = \frac{c}{a}$
$0 = a^2 - b^2$ $\quad e = \frac{0}{a}$
$b^2 = a^2$
$b = a$ $\quad e = 0$
The figure is a circle.

4. In an ellipse, $b^2 = a^2 - c^2$ and $\frac{c}{a} = e$.
$\frac{c}{a} = e$ $\quad b^2 = a^2 - c^2$
$c = ae$ $\quad b^2 = a^2 - a^2e^2$
$c^2 = a^2e^2$ $\quad b^2 = a^2(1 - e^2)$

5. Shanice; an equation with only one squared term cannot be the equation of an ellipse.

6. center: $(h, k) = (-7, 0)$
$a = |0 - 6|$ or 6
$b = |-7 - (-4)|$ or 3
$\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1$
$\frac{(y - 0)^2}{6^2} + \frac{[x - (-7)]^2}{3^2} = 1$
$\frac{y^2}{36} + \frac{(x + 7)^2}{9} = 1$
$c = \sqrt{a^2 - b^2}$ $\quad$ foci: $(h, k \pm c) = (-7, 0 \pm 3\sqrt{3})$
$c = \sqrt{6^2 - 3^2}$ $\quad = (-7, \pm 3\sqrt{3})$
$c = 3\sqrt{3}$

7. center: $(h, k) = (0, 0)$

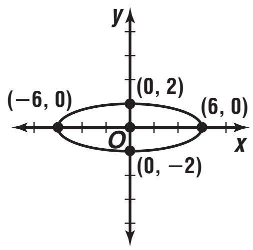
$a^2 = 36 \qquad b^2 = 4 \qquad c = \sqrt{a^2 - b^2}$

$a = \sqrt{36}$ or $6 \qquad b = \sqrt{4}$ or $2 \qquad c = \sqrt{36 - 4}$ or $4\sqrt{2}$

foci: $(h \pm c, k) = (0 \pm 4\sqrt{2}, 0)$ or $(\pm 4\sqrt{2}, 0)$

major axis vertices: $(h \pm a, k) = (0 \pm 6, 0)$ or $(\pm 6, 0)$

minor axis vertices: $(h, k \pm b) = (0, 0 \pm 2)$ or $(0, \pm 2)$



8. center: $(h, k) = (0, 4)$

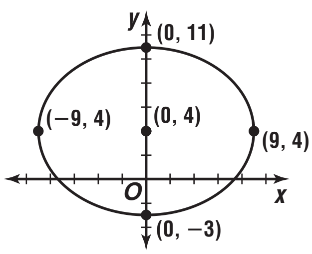
$a^2 = 81 \qquad b^2 = 49 \qquad c = \sqrt{a^2 - b^2}$

$a = \sqrt{81}$ or $9 \qquad b = \sqrt{49}$ or $7 \qquad c = \sqrt{81 - 49}$ or $4\sqrt{2}$

foci: $(h \pm c, k) = (0 \pm 4\sqrt{2}, 4)$ or $(\pm 4\sqrt{2}, 4)$

major axis vertices: $(h \pm a, k) = (0 \pm 9, 4)$ or $(\pm 9, 4)$

minor axis vertices: $(h, k \pm b) = (0, 4 \pm 7)$ or $(0, 11), (0, -3)$



9.
$$25x^2 + 9y^2 + 100x - 18y = 116$$
$$25(x^2 + 4x + ?) + 9(y^2 - 2y + ?) = 116 + ? + ?$$
$$25(x^2 + 4x + 4) + 9(y^2 - 2y + 1) = 116 + 25(4) + 9(1)$$
$$25(x + 2)^2 + 9(y - 1)^2 = 225$$
$$\frac{(x + 2)^2}{9} + \frac{(y - 1)^2}{25} = 1$$

center: $(h, k) = (-2, 1)$

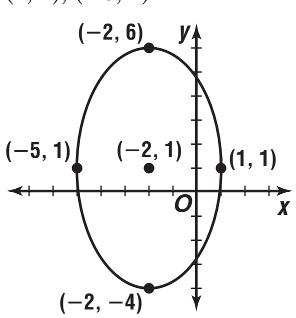
$a^2 = 25 \qquad b^2 = 9 \qquad c = \sqrt{a^2 - b^2}$

$a = \sqrt{25}$ or $5 \qquad b = \sqrt{9}$ or $3 \qquad c = \sqrt{25 - 9}$ or 4

foci: $(h, k \pm c) = (-2, 1 \pm 4)$ or $(-2, 5), (-2, -3)$

major axis vertices: $(h, k \pm a) = (-2, 1 \pm 5)$ or $(-2, 6), (-2, -4)$

minor axis vertices: $(h \pm b, k) = (-2 \pm 3, 1)$ or $(1, 1), (-5, 1)$



10.
$$9x^2 + 4y^2 - 18x + 16y = 11$$
$$9(x^2 - 2x + ?) + 4(y^2 + 4y + ?) = 11 + ? + ?$$
$$9(x^2 - 2x + 1) + 4(y^2 + 4y + 4) = 11 + 9(1) + (4)$$
$$9(x - 1)^2 + 4(y + 2)^2 = 36$$
$$\frac{(x - 1)^2}{4} + \frac{(y + 2)^2}{9} = 36$$

center: $(h, k) = (1, -2)$

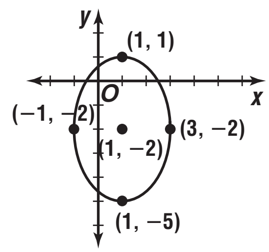
$a^2 = 9 \qquad b^2 = 4 \qquad c = \sqrt{a^2 - b^2}$

$a = \sqrt{9}$ or $3 \qquad b = \sqrt{4}$ or $2 \qquad c = \sqrt{9 - 4}$ or $\sqrt{5}$

foci: $(h, k \pm c) = \left(1, -2 \pm \sqrt{5}\right)$

major axis vertices: $(h, k \pm a) = (1, -2 \pm 3)$ or $(1, 1), (1, -5)$

minor axis vertices: $(h + b, k) = (1 \pm 2, -2)$ or $(3, -2), (-1, -2)$



11. center: $(h, k) = (-2, -3)$

$a = \frac{8}{2}$ or 4

$b = \frac{2}{2}$ or 1

$$\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1$$
$$\frac{[y - (-3)]^2}{4^2} + \frac{[x - (-2)]^2}{1^2} = 1$$
$$\frac{(y + 3)^2}{16} + \frac{(x + 2)^2}{1} = 1$$

12. The major axis contains the foci and it is located on the x-axis.

center: $(h, k) = \left(\frac{-1 + 1}{2}, \frac{0 + 0}{2}\right)$ or $(0, 0)$

$c = 1, a = 4$

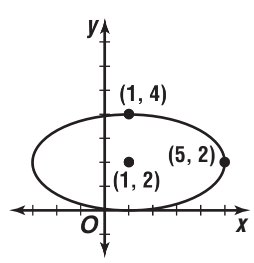
$c^2 = a^2 - b^2$

$1^2 = 4^2 - b^2$

$b^2 = 15$

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$
$$\frac{(x - 0)^2}{4^2} + \frac{(y - 0)^2}{15} = 1$$
$$\frac{x^2}{16} + \frac{y^2}{15} = 1$$

13. center: $(h, k) = (1, 2)$



The points at (1, 4) and (5, 2) are vertices of the ellipse.

$a = 4, b = 2$

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$
$$\frac{(x - 1)^2}{4^2} + \frac{(y - 2)^2}{2^2} = 1$$
$$\frac{(x - 1)^2}{16} + \frac{(y - 2)^2}{4} = 1$$

14. center: $(h, k) = (3, 1)$

$a = 6$ $\qquad$ $2 = c$

$e = \frac{c}{a}$ $\qquad$ $c^2 = a^2 - b^2$

$\frac{1}{3} = \frac{c}{6}$ $\qquad$ $2^2 = 6^2 - b^2$

$4 = 36 - b^2$

$b^2 = 32$

$\frac{(y-k)^2}{a^2} + \frac{(h-h)^2}{b^2} = 1$

$\frac{(y-1)^2}{6^2} + \frac{(x-3)^2}{32} = 1$

$\frac{(y-1)^2}{36} + \frac{(x-3)^2}{32} = 1$

15. The major axis contains the foci and is located on the x-axis.

center: $(h, k) = (0, 0)$

$c = 0.141732$

$a = \frac{1}{2}(3.048)$ or 1.524

$c^2 = a^2 - b^2$

$(0.141732)^2 = (1.524)^2 - b^2$

$0.020 = 2.323 - b^2$

$b^2 = 2.302$

$b \approx 1.517$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-0)^2}{1.524^2} + \frac{(y-0)^2}{1.517^2} = 1$

$\frac{x^2}{1.524^2} + \frac{y^2}{1.517^2} = 1$

Pages 638–641 Exercises

16. center: $(h, k) = (0, -5)$

$a = |0 - (-7)|$ or 7

$b = |-5 - 0|$ or 5

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-0)^2}{7^2} + \frac{[y-(-5)]^2}{5^2} = 1$

$\frac{x^2}{49} + \frac{(y+5)^2}{25} = 1$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{7^2 - 5^2}$ or $2\sqrt{6}$

foci: $(h \pm c, k) = (0 \pm 2\sqrt{6}, -5)$

$= (\pm 2\sqrt{6}, -5)$

17. center: $(h, k) = (-2, 0)$

$a = |-2 - 2|$ or 4

$b = |0 - 2|$ or 2

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\frac{[x-(-2)]^2}{4^2} + \frac{(y-0)^2}{2^2} = 1$

$\frac{(x+2)^2}{16} + \frac{y^2}{4} = 1$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{4^2 - 2^2}$ or $2\sqrt{3}$

foci: $(h \pm c, k) = (-2 \pm 2\sqrt{3}, 0)$

18. centers: $(h, k) = (-3, 4)$

$a = |4 - 12|$ or 8

$b = |-3 - 2|$ or 5

$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$

$\frac{(y-4)^2}{8^2} + \frac{[x-(-3)]^2}{5^2} = 1$

$\frac{(y-4)^2}{64} + \frac{(x+3)^2}{25} = 1$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{8^2 - 5^2}$ or $\sqrt{39}$

foci: $(h, k \pm c) = (3, 4 \pm \sqrt{39})$

19. center: $(h, k) = (-2, 1)$

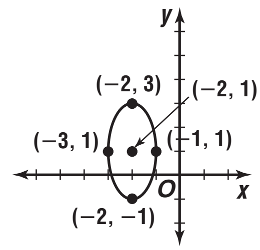
$a^2 = 4$ $\qquad$ $b^2 = 1$ $\qquad$ $c = \sqrt{a^2 - b^2}$

$a = \sqrt{4}$ or 2 $\qquad$ $b = \sqrt{1}$ or 1 $\qquad$ $c = \sqrt{4-1}$ or $\sqrt{3}$

foci: $(h, k \pm c) = (-2, 1 \pm \sqrt{3})$

major axis vertices: $(h, k \pm a) = (-2, 1 \pm 2)$ or $(-2, 3), (-2, -1)$

minor axis vertices: $(h \pm b, k) = (-2 \pm 1, 1)$ or



20. center: $(h, k) = (6, 7)$

$a^2 = 121$ $\qquad$ $b^2 = 100$

$a = \sqrt{121}$ or 11 $\qquad$ $b = \sqrt{100}$ or 10

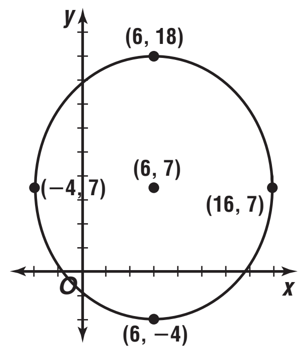
$c = \sqrt{a^2 - b^2}$

$c = \sqrt{121 - 100}$ or $\sqrt{21}$

foci: $(h, k \pm c) = (6, 7 \pm \sqrt{21})$

major axis vertices: $(h, k \pm a) = (6, 7 \pm 11)$ or $(6, 18), (6, -4)$

minor axis vertices: $(h \pm b, k) = (6 \pm 10, 7)$ or



21. center: $(h, k) = (4, -6)$

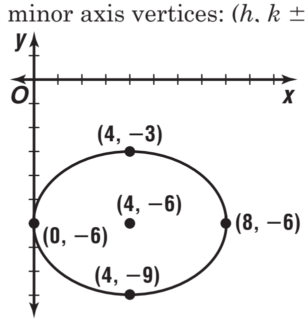
$a^2 = 16$ $\qquad$ $b^2 = 9$ $\qquad$ $c = \sqrt{a^2 - b^2}$

$a = \sqrt{16}$ or 4 $\qquad$ $b = \sqrt{9}$ or 3 $\qquad$ $c = \sqrt{16-9}$ or $\sqrt{7}$

foci: $(h \pm c, k) = (4 \pm \sqrt{7}, -6)$

major axis vertices: $(h \pm a, k) = (4 \pm 4, -6)$ or $(8, -6), (0, -6)$

minor axis vertices: $(h, k \pm b) = (4, -6 \pm 3)$ or



22. $(h, k) = (0, 0)$

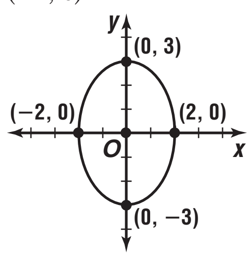
$a^2 = 9$ $\quad b^2 = 4$ $\quad c = \sqrt{a^2 - b^2}$

$a = \sqrt{9}$ or 3 $\quad b = \sqrt{4}$ or 2 $\quad c = \sqrt{9 - 4}$ or $\sqrt{5}$

foci: $(h, k \pm c) = \left(0, 0 \pm \sqrt{5}\right)$ or $\left(0, \pm \sqrt{5}\right)$

major axis vertices: $(h, k \pm a) = (0, 0 \pm 3)$ or $(0, \pm 3)$

minor axis vertices: $(h \pm b, k) = (0 \pm 2, 0)$ or $(\pm 2, 0)$



23.

$$4x^2 + y^2 - 8x + 6y + 9 = 0$$
$$4(x^2 - 2x + ?) + (y^2 + 6y + ?) = -9 + ? + ?$$
$$4(x^2 - 2x + 1) + (y^2 + 6y + 9) = -9 + 4(1) + 9$$
$$4(x - 1)^2 - (y + 3)^2 = 4$$
$$\frac{(x-1)^2}{1} + \frac{(y+3)^2}{4} = 1$$

center: $(h, k) = (1, -3)$

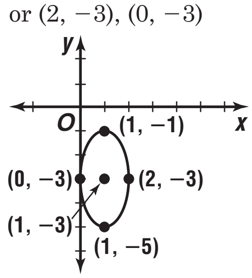
$a^2 = 4$ $\quad b^2 = 1$ $\quad c = \sqrt{a^2 - b^2}$

$a = \sqrt{4}$ or 2 $\quad b = \sqrt{1}$ or 1 $\quad c = \sqrt{4 - 1}$ or $\sqrt{3}$

foci: $(h, k \pm c) = \left(1, -3 \pm \sqrt{3}\right)$

major axis vertices: $(h, k \pm a) = (1, -3 \pm 2)$ or $(1, -1), (1, -5)$

minor axis vertices: $(h \pm b, k) = (1 \pm 1, -3)$ or $(2, -3), (0, -3)$



24.

$$16x^2 + 25y^2 - 96x - 200y = -144$$
$$16(x^2 - 6x + ?) + 25(y^2 - 8y + ?) = -144 + ? + ?$$
$$16(x^2 - 6x + 9) + 25(y^2 - 8y + 16) = -144 + 16(9) + 25(16)$$
$$16(x - 3)^2 + 25(y - 4)^2 = 400$$
$$\frac{(x-3)^2}{25} + \frac{(y-4)^2}{16} = 1$$

center: $(h, k) = (3, 4)$

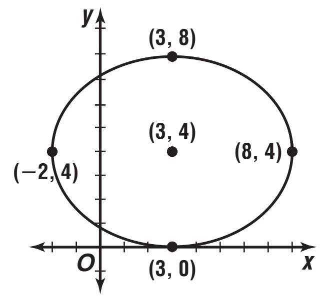
$a^2 = 25$ $\quad b^2 = 16$ $\quad c = \sqrt{a^2 - b^2}$

$a = \sqrt{25}$ or 5 $\quad b = \sqrt{16}$ or 4 $\quad c = \sqrt{25 - 16}$ or 3

foci: $(h \pm c, k) = (3 \pm 3, 4)$ or $(6, 4), (0, 4)$

major axis vertices: $(h \pm a, k) = (3 \pm 5, 4)$ or $(8, 4), (-2, 4)$

major axis vertices: $(h, k \pm b) = (3, 4 \pm 4)$ or $(3, 8), (3, 0)$



25.

$$3x^2 + y^2 + 18x - 2y + 4 = 0$$
$$3(x^2 + 6x + ?) + (y^2 - 2 + ?) = -4$$
$$3(x^2 + 6x + 9) + (y^2 - 2y + 1) = -4 + 3(9) + 1$$
$$3(x + 3)^2 + (y - 1)^2 = 24$$
$$\frac{(x+3)^2}{8} + \frac{(y-1)^2}{24} = 1$$

center: $(h, k) = (-3, 1)$

$a^2 = 24$ $\quad b^2 = 8$

$a = \sqrt{24}$ or $2\sqrt{6}$ $\quad b = \sqrt{8}$ or $2\sqrt{2}$

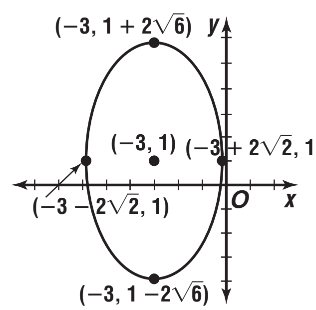
$c = \sqrt{a^2 - b^2}$

$c = \sqrt{24 - 8}$ or 4

foci: $(h, k \pm c) = (-3, 1 \pm 4)$ or $(-3, 5), (-3, -3)$

major axis vertices: $(h, k \pm a) = (-3, 1 \pm 2\sqrt{6})$

minor axis vertices: $(h \pm b, k) = (-3 \pm 2\sqrt{2}, 1)$



26.

$$6x^2 - 12x + 6y + 36y = 36$$
$$6(x^2 - 2x + ?) + 6(y^2 + 6y + ?) = 36$$
$$6(x^2 - 2x + 1) + 6(y^2 + 6y + 9) = 36 + 6(1) + 6(9)$$
$$6(x - 1)^2 + 6(y + 3)^2 = 96$$
$$\frac{(x-1)^2}{16} + \frac{(y+3)^2}{16} = 1$$

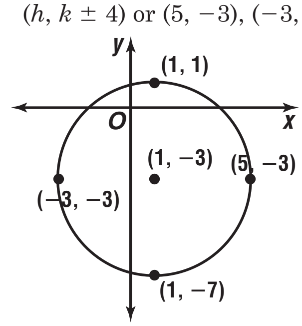
center: $(h, k) = (1, -3)$

$a^2 = 16$ $\quad b^2 = 16$ $\quad c = \sqrt{a^2 - b^2}$

$a = \sqrt{16}$ or 4 $\quad b = \sqrt{16}$ or 4 $\quad c = \sqrt{16 - 16}$ or 0

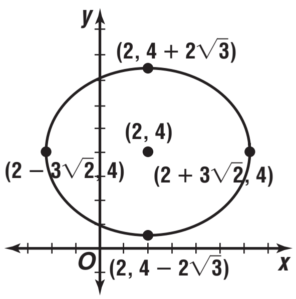
foci: $(h \pm c, k)$ or $(h, k \pm c) = (1, -3)$

Since $a = b = 4$, the vertices are $(h \pm 4, k)$ and $(h, k \pm 4)$ or $(5, -3), (-3, -3), (1, 1), (1, -7)$



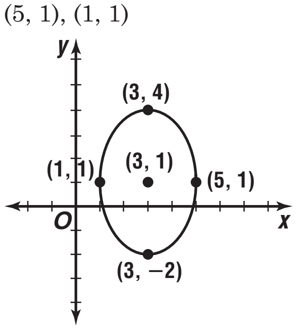
27. $18y^2 + 12x^2 - 144y - 48x = -120$
$18(y^2 - 8y + ?) + 12(x^2 - 4x + ?) = -120 + ? + ?$
$18(y^2 - 8y + 16) + 12(x^2 - 4x + 4) =$
$-120 + 18(16) + 12(4)$
$18(y - 4)^2 + 12(x - 2)^2 = 216$
$\frac{(y-4)^2}{12} + \frac{(x-2)^2}{18} = 1$

center: $(h, k) = (2, 4)$
$a^2 = 18$ $\quad b^2 = 12$
$a = \sqrt{18}$ or $3\sqrt{2}$ $\quad b = \sqrt{12}$ or $2\sqrt{3}$
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{18 - 12}$ or $\sqrt{6}$
foci: $(h \pm c, k) = (2 \pm \sqrt{6}, 4)$
major axis vertices: $(h \pm g, k) = (2 \pm 3\sqrt{2}, 4)$
minor axis vertices: $(h, k \pm b) = (2, 4 \pm 2\sqrt{3})$



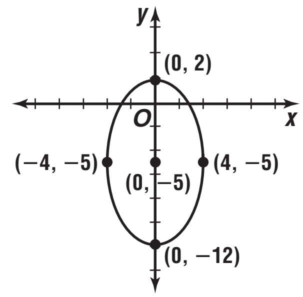
28. $4x^2 - 8y + 9x^2 - 54x + 49 = 0$
$4(y^2 - 2y + ?) + 9(x^2 - 6x + ?) = -49 + ? + ?$
$4(y^2 - 2y + 1) + 9(x^2 - 6x + 9) = -49 + 4(1) + 9(9)$
$4(y - 1)^2 + 9(x - 3)^2 = 36$
$\frac{(y-1)^2}{9} + \frac{(x-3)^2}{4} = 1$

center: $(h, k) = (3, 1)$
$a^2 = 9$ $\quad b^2 = 4$ $\quad c = \sqrt{a^2 - b^2}$
$a = \sqrt{9}$ or 3 $\quad b = \sqrt{4}$ or 2 $\quad c = \sqrt{9 - 4}$ or $\sqrt{5}$
foci: $(j, k \pm c) = (3, 1 \pm 5)$
major axis vertices: $(h, k \pm a) = (3, 1 \pm 3)$ or $(3, 4)$, $(3, 2)$
minor axis vertices: $(h \pm b, k) = (3 \pm 2, 1)$ or $(5, 1)$, $(1, 1)$



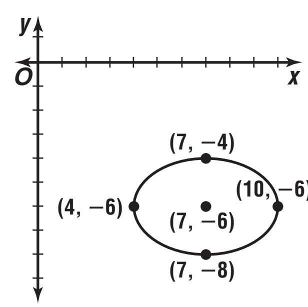
29. $49x^2 + 16y^2 + 160y - 384 = 0$
$49x^2 + 16(y^2 + 10y + ?) = 384 + ?$
$49x^2 + 16(y^2 + 10y + 25) = 384 + 16(25)$
$49x^2 + 16(y - 5)^2 = 784$
$\frac{x^2}{16} + \frac{(y+5)^2}{49} = 1$

center: $(h, k) = (0, -5)$
$a^2 = 49$ $\quad b^2 = 16$ $\quad c = \sqrt{a^2 - b^2}$
$a = \sqrt{49}$ or 7 $\quad b = \sqrt{16}$ or 4 $\quad c = \sqrt{49 - 16}$ or $\sqrt{33}$
foci: $(h, k \pm c) = (0, -5 \pm \sqrt{33})$
major axis vertices: $(h, k \pm a) = (0, -5 \pm 7)$ or $(0, 2)$, $(0, -12)$
minor axis vertices: $(h \pm b, k) = (0 \pm 4, -5)$ or $(\pm 4, -5)$



30. $9y^2 + 108y + 4x^2 - 56x = -484$
$9(y^2 + 12y + ?) + 4(x^2 - 14x + ?) = -484 + ? + ?$
$9(y^2 + 12y + 36) + 4(x^2 - 14x + 49) =$
$-484 + 9(36) + 4(49)$
$9(y + 6)^2 + 4(x - 7)^2 = 36$
$\frac{(y+6)^2}{4} + \frac{(x-7)^2}{9} = 1$

center: $(h, k) = (7, -6)$
$a^2 = 9$ $\quad b^2 = 4$ $\quad c = \sqrt{a^2 - b^2}$
$a = \sqrt{9}$ or 3 $\quad b = \sqrt{4}$ or 2 $\quad c = \sqrt{9 - 4}$ or $\sqrt{5}$
foci: $(h \pm c, k) = (7 \pm \sqrt{5}, -6)$
major axis vertices: $(h \pm a, k) = (7 \pm 3, -6)$ or $(10, -6)(4, -6)$
minor axis vertices: $(h, k \pm b) = (7, -6 \pm 2)$ or $(7, -4)$, $(7, -8)$



31. $a = 7, b = 5$
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$
$$\frac{[x-(-3)]^2}{7^2} + \frac{[y-(-1)]^2}{5^2} = 1$$
$$\frac{(x+3)^2}{49} + \frac{(y+1)^2}{25} = 1$$

32. The major axis contains the foci and it is located on the x-axis.

center: $(h, k) = \left(\frac{-2+2}{0}, \frac{0+0}{2}\right)$ or $(0, 0)$

$c = 2, a = 7$
$c^2 = a^2 - b^2$
$2^2 = 7^2 - b^2$
$b^2 = 45$
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
$\frac{(x-0)^2}{7^2} + \frac{(y-0)^2}{45} = 1$
$\frac{x^2}{49} + \frac{y^2}{45} = 1$

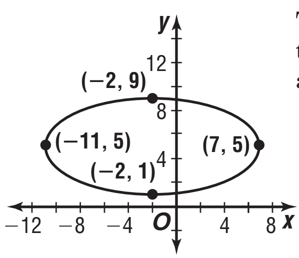
33. $b = \frac{3}{4}a$
$6 = \frac{3}{4}a$
$8 = a$
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
$\frac{(x-0)^2}{8^2} + \frac{(y-0)^2}{6^2} = 1$
$\frac{x^2}{64} + \frac{y^2}{36} = 1$

34. The major axis contains the foci and it is the vertical axis of the ellipse.

center: $(h, k) = \left(\frac{-1+(-1)}{2}, \frac{1+(-5)}{2}\right)$ or $(-1, -2)$

$c = |1 - k|$ $\qquad a = 2\sqrt{13}$
$c = |1 - (-2)|$ or 3
$c^2 = a^2 - b^2$
$3^2 = \left(2\sqrt{13}\right)^2 - b^2$
$b^2 = 52 - 9$
$b^2 = 43$
$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
$\frac{[y-(-2)]^2}{(2\sqrt{13})^2} + \frac{[x-(-1)]^2}{43} = 1$
$\frac{(y+2)^2}{52} + \frac{(x+1)^2}{43} = 1$

35.



The horizontal axis of the ellipse is the major axis.

enter: $(h, k) = \left(\frac{-11+7}{2}, \frac{5+5}{2}\right)$ or $(-2, 5)$

$h + a = 7$ $\qquad k + b = 9$
$-2 + a = 7$ $\qquad 5 + b = 9$
$a = 9$ $\qquad b = 4$
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
$\frac{[x-(-2)]^2}{9^2} + \frac{(y-5)^2}{4^2} = 1$
$\frac{(x+2)^2}{81} + \frac{(y-5)^2}{16} = 1$

36. The major axis contains the foci and it is the vertical axis of the ellipse.

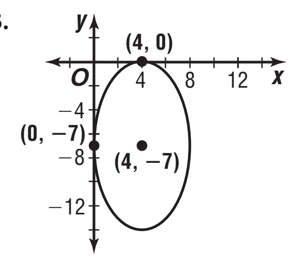
$c = \frac{5-(-1)}{2}$ or 3

center: $(h, k) = \left(\frac{1+1}{2}, \frac{-1+5}{2}\right)$ or $(1, 2)$

$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$ $\qquad c^2 = a^2 - b^2$
$\frac{(2-2)^2}{a^2} + \frac{(y-1)^2}{b^2} = 1$ $\qquad 3^2 = a^2 - 9$
$\frac{0^2}{a^2} + \frac{3^2}{b^2} = 1$ $\qquad 18 = a^2$
$\frac{9}{b^2} = 1$
$9 = b^2$
$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
$\frac{(y-2)^2}{18} + \frac{(x-1)^2}{9} = 1$

37. $\frac{1}{2} = \frac{c}{a}$ $\qquad b^2 = a^2 - c^2$
$\frac{a}{2} = c$ $\qquad b^2 = 10^2 - 5^2$
$\frac{10}{2} = c$ $\qquad b^2 = 75$
$5 = c$
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
$\frac{(x-0)^2}{10^2} + \frac{(y-0)^2}{75} = 1$
$\frac{x^2}{100} + \frac{y^2}{75} = 1$

38.



tangent vertices: (4, 0), (0, −7)
$a = |-7 - 0|$ or 7
$b = |4 - 0|$ or 4

$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
$\frac{[y-(-7)]}{7^2} + \frac{(x-4)^2}{4^2} = 1$
$\frac{(y+7)^2}{49} + \frac{(x-4)^2}{16} = 1$

39. $b^2 = a^2(1 - e^2)$
$b^2 = 2^2\left[1 - \left(\frac{3}{4}\right)^2\right]$
$b^2 = \frac{28}{16}$ or 1.75

Case 1: Horizontal axis is major axis.
$\frac{(x-h)^2}{a^2} + \frac{(y-k)}{b^2} = 1$
$\frac{(x-0)^2}{2^2} + \frac{(y-0)^2}{1.75} = 1$
$\frac{x^2}{4} + \frac{y^2}{1.75} = 1$

Case 2: Vertical axis is major axis.
$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
$\frac{(y-0)^2}{2^2} + \frac{(x-0)^2}{1.75} = 1$
$\frac{y^2}{4} + \frac{x^2}{1.75} = 1$

40. The major axis contains the foci and it is the horizontal axis of the ellipse.

center: $(h, k) = \left(\frac{3+1}{2}, \frac{5+5}{2}\right)$ or $(2, 5)$

foci: $(3, 5) = (h + c, k)$

$3 = h + c$

$3 = 2 + c$

$1 = c$

$e = \frac{c}{a}$ $\qquad b^2 = a^2(1 - e^2)$

$0.25 = \frac{1}{a}$ $\qquad b^2 = 4^2(1 - 0.25^2)$

$a = 4$ $\qquad b^2 = 15$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-2)^2}{4^2} + \frac{(y-5)^2}{15} = 1$$
$$\frac{(x-2)^2}{16} + \frac{(y-15)^2}{15} = 1$$

41. $a = \frac{20}{2}$ or 10

$b^2 = a^2(1 - e^2)$

$b^2 = 10^2\left[1 - \left(\frac{7}{10}\right)^2\right]$ or 51

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$
$$\frac{(y-0)^2}{10^2} + \frac{(x-3)^2}{51} = 1$$
$$\frac{y^2}{100} + \frac{(x-3)^2}{51} = 1$$

42. focus: $(1, -1 + \sqrt{5}) = (h, k + c)$

$-1 + \sqrt{5} = k + c$

$-1 + \sqrt{5} = -1 + c$

$\sqrt{5} = c$

$e = \frac{c}{a}$ $\qquad b^2 = a^2(1 - e^2)$

$\frac{\sqrt{5}}{3} = \frac{\sqrt{5}}{a}$ $\qquad b^2 = 3^2\left[1 - \left(\frac{\sqrt{5}}{3}\right)^2\right]$

$a = 3$ $\qquad b^2 = 4$

major axis: vertical axis

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$
$$\frac{[y-(-1)]^2}{3^2} + \frac{(x-1)^2}{4} = 1$$
$$\frac{(y+1)^2}{9} + \frac{(x-1)^2}{4} = 1$$

43. $x^2 + 4y^2 - 6x + 24y = -41$

$(x^2 - 6x + ?) + 4(y^2 + 6y + ?) = -41 + ? + ?$

$(x^2 - 6x + 9) + 4(y^2 + 6y + 9) = -41 + 9 + 4(9)$

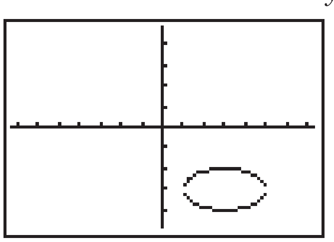
$(x - 3)^2 + 4(y + 3)^2 = 4$

$4(y + 3)^2 = 4 - (x - 3)^2$

$(y + 3)^2 = \frac{4 - (x-3)^2}{4}$

$y + 3 = \pm\sqrt{\frac{4 - (x-3)^2}{4}}$

$y = \pm\sqrt{\frac{4 - (x-3)^2}{4}} - 3$



vertices: $(5, -3)$, $(1, -3)$, $(3, -2)$, $(3, -4)$

$[-7.28, 7.28]$ scl:1 by $[-4.8, 4.8]$ scl:1

44. $4x^2 + y^2 - 8x - 2y = -1$

$4(x^2 - 2x + ?) + (y^2 - 2y + ?) = -1 + ? + ?$

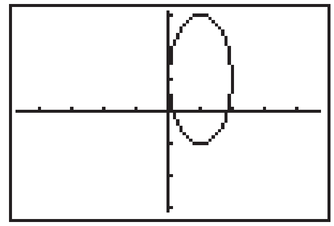
$4(x^2 - 2x + 1) + (y^2 - 2y + 1) = -1 + 4(1) + 1$

$4(x - 1)^2 + (y - 1)^2 = 4$

$(y - 1)^2 = 4 - 4(x - 1)^2$

$y - 1 = \pm\sqrt{-4 - 4(x-1)^2}$

$y = \pm\sqrt{4 - 4(x-1)^2} + 1$



Vertices: $(0, 1)$, $(2, 1)$, $(1, -1)$, $(1, 3)$

$[-4.7, 4.7]$ scl:1 by $[-3.1, 3.1]$ scl:1

45. $4x^2 + 9y^2 - 16x + 18y = 11$

$4(x^2 - 4x + ?) + 9(y^2 + 2y + ?) = 11 + ? + ?$

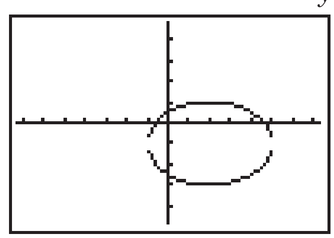
$4(x^2 - 4x + 4) + 9(y^2 + 2y + 1) = 11 + 4(4) + 9(1)$

$4(x - 2)^2 + 9(y + 1)^2 = 36$

$9(y + 1)^2 = 36 - 4(x - 2)^2$

$y + 1 = \pm\sqrt{\frac{36 - 4(x-2)^2}{9}}$

$y = \pm\sqrt{\frac{36 - 4(x-2)^2}{9}} - 1$



Vertices: $(-1, -1)$, $(5, -1)$, $(2, -3)$, $(2, 1)$

$[-7.28, 7.28]$ scl:1 by $[-4.8, 4.8]$ scl:1

46. $25y^2 + 16x^2 - 150y + 32x = 159$

$25(y^2 - 6y + ?) + 16(x^2 + 2x - ?) = 159 + ? + ?$

$25(y^2 - 6y + 9) + 16(x^2 - 2x + 1) =$

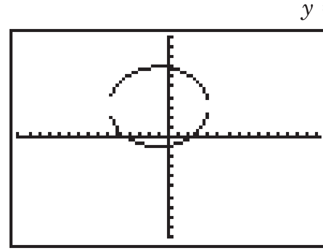
$159 + 25(9) + 16(1)$

$25(y - 3)^2 + 16(x + 1)^2 = 400$

$25(y - 3)^2 = 400 - 16(x + 1)^2$

$(y - 3) = \pm\sqrt{\frac{400 - 16(x+1)^2}{25}}$

$y = \pm\sqrt{\frac{400 - 16(x+1)^2}{5}} + 3$



Vertices: $(4, 3)$, $(-6, 3)$, $(-1, 7)$, $(-1, -1)$

$[-15.16, 15.16]$ scl:1 by $[-10, 10]$ scl:1

47. The target ball should be placed opposite the pocket, $\sqrt{5}$ feet from the center along the major axis of the ellipse. The cue ball can be placed anywhere on the side opposite the pocket. The ellipse has a semi-major axis of length 3 feet and a semi-minor axis of length 2 feet. Using the equation $c^2 = a^2 - b^2$, the focus of the ellipse is found to be $\sqrt{5}$ feet from the center of the ellipse. Thus the hole is located at one focus of the ellipse. The reflective properties of an ellipse should insure that a ball placed $\sqrt{5}$ feet from the center of the ellipse and hit so that it rebounds once off the wall should fall into the pocket at the other focus of the ellipse.

48. A horizontal line; see students' work.

49a. $a = \frac{96}{2}$ or 48

$b = \frac{46}{2}$ or 23

$(h, k) = (0, 0)$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-0)^2}{48^2} + \frac{(y-0)^2}{23^2} = 1$$
$$\frac{x^2}{2304} + \frac{y^2}{529} = 1$$

49b. $c = \sqrt{a^2 - b^2}$

$c = \sqrt{2304 - 529}$

$c \approx 42.13$

He could have stood at a focal point, about 42 feet on either side of the center along the major axis.

49c. The distance between the focal points is $2c$.

$2c = 2(42)$

$= 84$

about 84 ft

50a. $x^2 + y^2 = r^2$

$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$ $\qquad A = \pi r^2$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\qquad A = \pi \cdot r \cdot r$

$A = \pi \cdot a \cdot b$

$A = \pi ab$

50b. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

$a^2 = 9$ $\qquad b^2 = 4$

$a = 3$ $\qquad b = 2$

$A = \pi ab$

$A = \pi(3)(2)$

$A = 6\pi$ units^2

51. If (x, y) is a point on the ellipse, then show that $(-x, -y)$ is also on the ellipse.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
$$\frac{(-x)^2}{a^2} + \frac{(-y)^2}{b^2} = 1$$
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Thus, $(-x, -y)$ is also a point on the ellipse and the ellipse is symmetric with respect to the origin.

52a. $a = \frac{8}{2}$ or 4

$b = 3$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{4^2 - 3^2}$

$c = \sqrt{7}$

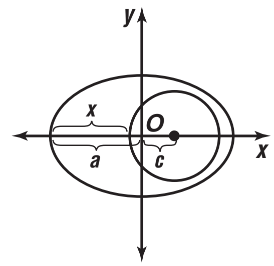
foci: $(h \pm c, 0) = (0 \pm \sqrt{7}, 0)$ or $(\pm\sqrt{7}, 0)$

The thumbtacks should be placed $(\pm\sqrt{7}, 0)$ from the center of the arch.

52b. With the string anchored by thumbtacks at the foci of the arch and held taunt by a pencil, the sum of the distances from each thumbtack to the pencil will remain constant.

53a. GOES 4; its eccentricity is closest to 0.

53b.



$\frac{c}{a} = e$

$\frac{c}{6955} = 0.052$

$c = 361.66$

[figure not drawn to scale]

$x = a + c -$ Earth's radius

$x = 6955 + 361.66 - 6357$

$x = 959.66$

$x \approx 960$ km

54.

$$x^2 + y^2 + Dx + Ey + F = 0$$
$$0^2 + (-9)^2 + D(0) + E(-9) + F = 0 \Rightarrow$$
$$-9E + F = -81$$
$$7^2 + (-2)^2 + D(7) + E(-2) + F = 0 \Rightarrow$$
$$7D - 2E + F = -53$$
$$(-5)^2 + (-10)^2 + D(-5) + E(-10) + F = 0 \Rightarrow$$
$$-5D - 10E + F = -125$$

$$\begin{array}{r} -9E + F = -81 \\ (-1)(7D - 2E + F) = (-1)(-53) \\ \hline -7D - 7E \quad = -28 \\ D + E = 4 \end{array}$$

$$\begin{array}{r} 7D - 2E + F = -53 \\ (-1)(-5D - 10E + F) = (-1)(-125) \\ \hline 12D + 8E \quad = 72 \end{array}$$

$$\begin{array}{r} (-8)(D + E) = (8)(4) \\ 12D + 8E = 72 \\ \hline 4D \quad = 40 \\ D = 10 \end{array}$$

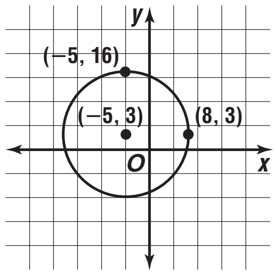
$D + E = 4$ $\qquad -9E + F = -81$

$10 + E = 4$ $\qquad -9(-6) + F = -81$

$E = -6$ $\qquad F = -135$

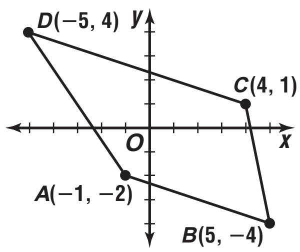
$$x^2 + y^2 + Dx + Ey + F = 0$$
$$x^2 + y^2 + 10x - 6y - 135 = 0$$
$$(x^2 + 10x + 25) + (y^2 - 6y + 9) = 135 + 25 + 9$$
$$(x + 5)^2 + (y - 3)^2 = 169$$

center: $(h, k) = (-5, 3)$
radius: $r^2 = 169$
$r = 13$



55. Graph the quadrilateral with vertices $A(-1, -2)$, $B(5, -4)$, $C(4, 1)$, and $D(-5, 4)$.

A quadrilateral is a parallelogram if one pair of opposite sides are parallel and congruent.



slope of $\overline{DA}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{-1 - (-5)} = \frac{-6}{4} \text{ or } \frac{3}{2}$$

slope of $\overline{CB}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 1}{5 - 4} = -\frac{5}{1}$$

The slopes are not equal, so $\overline{DA} \nparallel \overline{CB}$. The quadrilateral is not a parallelogram; no.

56. $\cos 2\theta = 1 - 2\sin^2\theta$
$\cos 2\theta = 1 - 2\left(\frac{7}{8}\right)^2$
$\cos 2\theta = \frac{-34}{64}$ or $-\frac{17}{32}$

57. $|A| = 4$
$A = \pm 4$
$-\frac{c}{k} = 20°$
$-\frac{c}{2} = 20°$
$c = -40°$

$\frac{2\pi}{k} = 180$
$k = 2$
$h = 0$

$y = A\cos(kx + c) + h$
$y = \pm 4\cos[2x + (-40°)] + 0$
$y = \pm 4\cos(2x - 40°)$

58. $A = 180° - (121° \, 32' + 42° \, 5')$ or $16° \, 23'$

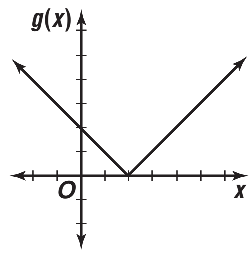
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{4.1}{\sin 16° \, 23'} = \frac{b}{\sin 42° \, 5'}$$
$$\frac{4.1 \sin 42° \, 5'}{\sin 16° \, 23'} = b$$
$$9.7 \approx b$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{4.1}{\sin 16° \, 23'} = \frac{c}{\sin 121° \, 32'}$$
$$\frac{4.1 \sin 121° \, 32'}{\sin 16° \, 23'} = c$$
$$12.4 \approx c$$

59. $P(x) = x^4 - 4x^3 - 2x^2 - 1$
$P(5) = 5^4 - 4(5)^3 - 2(5)^2 - 1$
$P(5) = 74$
$P(5) \neq 0$; no, the binomial is not a factor of the polynomial.

60. Let $h = 0.1$.
$x - h = x - 0.1$
$= -1 - 0.1$ or -1.1
$f(x - 0.1) = f(-1.1)$
$= (-1.1)^2 + 4(-1.1) - 12$
$= -15.19$
$x + h = x + 0.1$
$= -1 + 0.1$ or -0.9
$f(x + 0.1) = f(-0.9)$
$= (-0.9)^2 + 4(-0.9) - 12$
$= -14.79$
$f(x) = -16$
$f(x) < f(x - 0.1)$ and $f(x) < f(x - 0.1)$, so the point is a location of a minimum.

61. The graph of the parent function $g(x) = |x|$ is translated 2 units right.



62. Initial location: (2, 0)

$$\text{Rot}_{90} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \text{ or } (0, 2)$$

$$\text{Rot}_{80} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \text{ or } (-2, 0)$$

$$\text{Rot}_{270} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix} \text{ or } (0, -2)$$

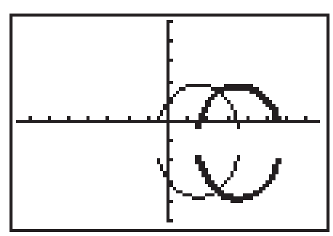
63. $m\angle QTS + m\angle TSR = 180$
$a + b + c + d = 180$
$b + b + c + c = 180$
$2b + 2c = 180$
$b + c = 90$

$p + b + c = 180$
$p + 90 = 180$
$p = 90$

The correct choice is *C*.

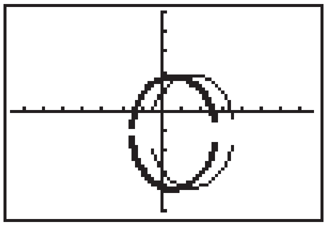
Page 641 Graphing Calculator Exploration

1. Sample answer: The graph will shift 4 units to the right.



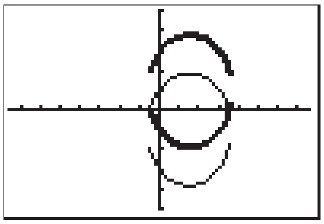
$[-15.16, 15.16]$ scl:2 by $[-10, 10]$ scl:2

2. Sample answer: The graph will shift 4 units to the left.



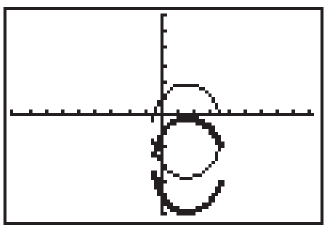
$[-15.16, 15.16]$ scl:2 by $[-10, 10]$ scl:2

3. Sample answer: The graph will shift 4 units up.



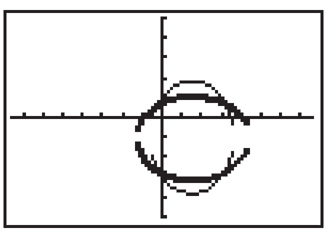
$[-15.16, 15.16]$ scl:2 by $[-10, 10]$ scl:2

4. Sample answer: The graph will shift 4 units down.



$[-18.19, 18.19]$ scl:2 by $[-12, 12]$ scl:2

5. Sample answer: The graph will rotate 90°.



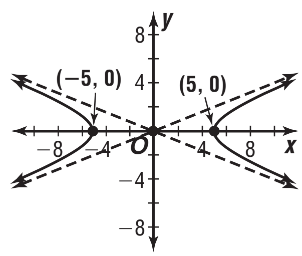
$[-15.16, 15.16]$ scl:2 by $[-10, 10]$ scl:2

6. For $(x + c)$, the graph will shift c units to the left. For $(x - c)$, the graph will shift c units to the right.
7. For $(y + c)$, the graph will shift c units down. For $(y - c)$, the graph will shift c units up.
8. The graph will rotate 90°.

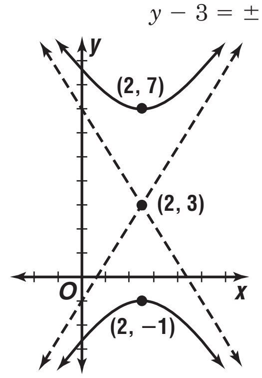
10-4 Hyperbolas

Pages 649–650 Check For Understanding

1. The equations of both hyperbolas and ellipses have x^2 terms and y^2 terms. In an ellipse, the terms are added and in a hyperbola these terms are subtracted.
2. transverse axis: vertical
$2a = 4$
$a = 2$
An equation in standard form of the hyperbola must have $\frac{y^2}{2^2}$ or $\frac{y^2}{4}$ as the first term; b.
3. $e = \frac{c}{a}$, so $ae = c$ and $a^2e^2 = c^2$.
Since $c^2 = a^2 + b^2$ we have
$$a^2e^2 = a^2 + b^2$$
$$a^2e^2 - a^2 = b^2$$
$$a^2(e^2 - 1) = b^2$$
4. With the equation in standard form, if the first expression contains "x", the transverse axis is horizontal. If the first expression contains "y", the transverse axis is vertical.
5. center: $(h, k) = (0, 0)$
$a^2 = 25$ $\quad b^2 = 4$ $\quad c = \sqrt{a^2 + b^2}$
$a = 5$ $\quad b = 2$ $\quad c = \sqrt{25 + 4}$ or $\sqrt{29}$
transverse axis: horizontal
foci: $(h + c, k) = (0 \pm \sqrt{29}, 0)$ or $(\pm\sqrt{29}, 0)$
vertices: $(h \pm a, k) = (0 \pm 5, 0)$ or $(\pm 5, 0)$
asymptotes: $y - k = \pm\frac{b}{a}(x - h)$
$$y - 0 = \pm\frac{2}{5}(x - 0)$$
$$y = \pm\frac{2}{5}x$$



6. center: $(h, k) = (2, 3)$
$a^2 = 16$ $\quad b^2 = 4$ $\quad c = \sqrt{a^2 + b^2}$
$a = \sqrt{16}$ or 4 $\quad b = \sqrt{4}$ or 2 $\quad c = \sqrt{16 + 4}$ or $2\sqrt{5}$
transverse axis: vertical
foci: $(h, k \pm c) = \left(2, 3 \pm 2\sqrt{5}\right)$
vertices: $(h, k \pm a) = (2, 3 \pm 4)$ or $(2, 7), (2, -1)$
asymptotes: $y - k = \pm\frac{a}{b}(x - h)$
$$y - 3 = \pm\frac{4}{2}(x - 2)$$
$$y - 3 = \pm 2(x - 2)$$



7. $y^2 - 5x^2 + 20x = 50$

$y^2 - 5(x^2 - 4x + ?) = 50 + ?$

$y^2 - 5(x^2 - 4x + 4) = 50 + (-5)(4)$

$y^2 - 5(x - 2)^2 = 30$

$\frac{y^2}{30} - \frac{(x-2)^2}{6} = 1$

$x - h = x - 2 \qquad y - k = y$

$h = 2 \qquad k = 0$

center: $(h, k) = (2, 0)$

$a^2 = 30 \qquad b^2 = 6 \qquad c = \sqrt{a^2 - b^2}$

$a = \sqrt{30} \qquad b = \sqrt{6} \qquad c = \sqrt{30 + 6}$ or 6

transverse axis: vertical

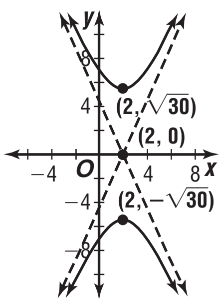
foci: $(h, k \pm c) = (2, 0 \pm 6)$ or $(2, \pm 6)$

vertices: $(h, k \pm a) = \left(2, 0 \pm \sqrt{30}\right)$ or $\left(2, \pm\sqrt{30}\right)$

asymptotes: $y - k = \pm \frac{a}{b}(x - h)$

$y - 0 = \pm\frac{\sqrt{30}}{\sqrt{6}}(x - 2)$

$y = \pm\sqrt{5}(x - 2)$



8. center: $(h, k) = (0, 5)$

transverse axis: horizontal

$a = 5, b = 3$

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-0)^2}{5^2} - \frac{(y-5)^2}{3^2} = 1$

$\frac{x^2}{25} - \frac{(y-5)^2}{9} = 1$

9. $c = -9$

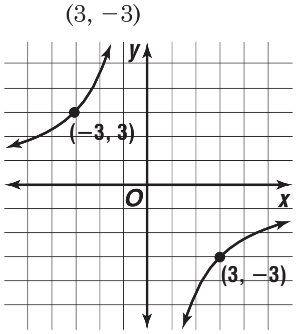
quadrants: II and IV

transverse axis: $y = -x$

vertices: $xy = -9 \qquad xy = -9$

$3(-3) = -9 \qquad -3(3) = -9$

$(3, -3) \qquad (-3, 3)$



10. center: $(h, k) = (1, -4)$

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-1)^2}{5^2} - \frac{[y-(-4)]^2}{2^2} = 1$

$\frac{(x-1)^2}{25} - \frac{(y+4)^2}{4} = 1$

11. $2b = 6$

$b = 3$

center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3+3}{2}, \frac{4-0}{2}\right)$

$= (3, 2)$

transverse axis: vertical

$a = |4 - 2|$ or 2

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$\frac{(y-2)^2}{2^2} - \frac{(x-3)^2}{3^2} = 1$

$\frac{(y-2)^2}{4} - \frac{(x-3)^2}{9} = 1$

12. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0+0}{2}, \frac{6+(-6)}{2}\right)$

$= (0, 0)$

transverse axis: vertical

c = distance from center to a focus

$= |0 - 6|$ or 6

$b^2 = c^2 - a^2 \qquad b^2 = a^2$

$a^2 = c^2 - a^2 \qquad b^2 = 18$

$2a^2 = c^2$

$a^2 = \frac{c^2}{2}$

$a^2 = \frac{6^2}{2}$ or 18

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$\frac{(y-0)^2}{18} - \frac{(x-0)^2}{18} = 1$

$\frac{y^2}{18} - \frac{x^2}{18} = 1$

13. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{10+(-10)}{2}, \frac{0+0}{2}\right)$

$= (0, 0)$

transverse axis: horizontal

c = distance from center to a focus

$= |10 - 0|$ or 10

$e = \frac{c}{a} \qquad b^2 = c^2 - a^2$

$\frac{5}{3} = \frac{10}{a} \qquad b^2 = 10^2 - 6^2$

$b^2 = 64$

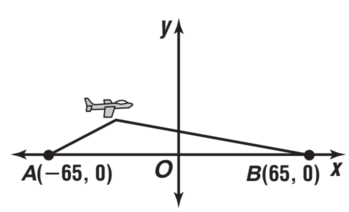
$a = 6$

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-0)^2}{6^2} - \frac{(y-0)^2}{64} = 1$

$\frac{x^2}{36} - \frac{y^2}{64} = 1$

14a. The origin is located midway between stations A and B; $(h, k) = (0, 0)$. The stations are located at the foci, so $2c = 130$ or $c = 65$.



The difference of the distances from the plane to each station is 50 miles.

$50 = 2a$ (Definition of hyperbola)

$25 = a$

$b^2 = c^2 - a^2$

$b^2 = 65^2 - 25^2$

$b^2 = 3600$

transverse axis: horizontal

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-0)^2}{25^2} - \frac{(y-0)^2}{3600} = 1$

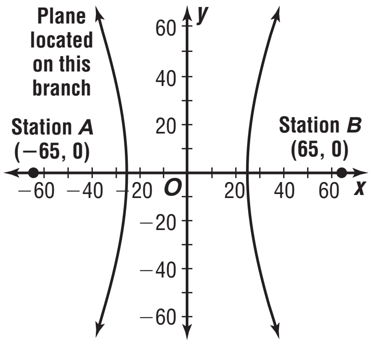
$\frac{x^2}{625} - \frac{y^2}{3600} = 1$

14b. Vertices: $(h \pm a, k) = (0 \pm 25, 0)$ or $(\pm 25, 0)$

asymptotes: $y - k = \pm\frac{b}{a}(x - h)$

$y - 0 = \pm\frac{\sqrt{3600}}{25}(x - 0)$

$y = \pm\frac{12}{5}x$



14c. Let $y = 6$.

$\frac{x^2}{625} - \frac{y^2}{3600} = 1$

$\frac{x^2}{625} - \frac{6^2}{3600} = 1$

$\frac{x^2}{625} - \frac{36}{3600} = 1$

$\frac{x^2}{625} = 1 + \frac{36}{3600}$

$\frac{x^2}{625} = 1.01$

$x^2 = 625(1.01)$

$x^2 = 631.25$

$x = \pm\sqrt{631.25}$

$x \approx \pm 25.1$

Since the phase is closer to station A than station B, use the negative value of x to locate the ship at $(-25.1, 6)$.

Pages 650–652 Exercises

15. center: $(h, k) = (0, 0)$

$a^2 = 100$ $\quad$ $b^2 = 16$

$a = \sqrt{100}$ or 10 $\quad$ $b = \sqrt{16}$ or 4

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{100 + 16}$ or $2\sqrt{29}$

transverse axis: horizontal

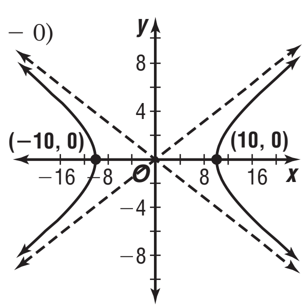
foci: $(h \pm c, k) = (0 \pm 2\sqrt{29}, 0)$ or $(\pm 2\sqrt{29}, 0)$

vertices: $(h \pm a, k) = (0 \pm 10, 0)$ or $(\pm 10, 0)$

asymptotes: $y - k = \pm\frac{b}{a}(x - h)$

$y - 0 = \pm\frac{4}{10}(x - 0)$

$y = \pm\frac{2}{5}x$



16. center: $(h, k) = (0, 5)$

$a^2 = 9$ $\quad$ $b^2 = 81$

$a = \sqrt{9}$ or 3 $\quad$ $b = \sqrt{81}$ or 9

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{9 + 81}$ or $3\sqrt{10}$

transverse axis: horizontal

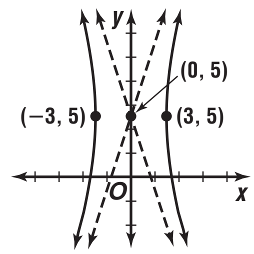
foci: $(h \pm c, k) = (0 \pm 3\sqrt{10}, 5)$ or $(\pm 3\sqrt{10}, 5)$

vertices: $(h \pm a, k) = (0 \pm 3, 5)$ or $(\pm 3, 5)$

asymptotes: $y - k = \pm\frac{b}{a}(x - h)$

$y - 5 = \pm\frac{9}{3}(x - 0)$

$y - 5 = \pm 3x$



17. center: $(h, k) = (0, 0)$

$a^2 = 4$ $\quad$ $b^2 = 49$

$a = \sqrt{4}$ or 2 $\quad$ $b = \sqrt{49}$ or 7

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{4 + 49}$ or $\sqrt{53}$

transverse axis: horizontal

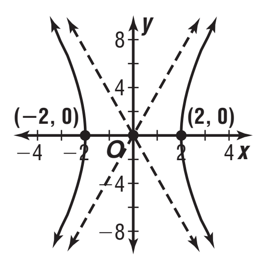
foci: $(h \pm c, k) = (0 \pm \sqrt{53}, 0)$ or $(\pm\sqrt{53}, 0)$

vertices: $(h \pm a, k) = (0 \pm 2, 0)$ or $(\pm 2, 0)$

asymptotes: $y - k = \pm\frac{b}{a}(x - h)$

$y - 0 = \pm\frac{7}{2}(x - 0)$

$y = \pm\frac{7}{2}x$



18. center: $(h, k) = (-1, 7)$

$a^2 = 64$ $\quad$ $b^2 = 4$

$a = \sqrt{64}$ or 8 $\quad$ $b = \sqrt{4}$ or 2

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{64 + 4}$ or $2\sqrt{17}$

transverse axis: vertical

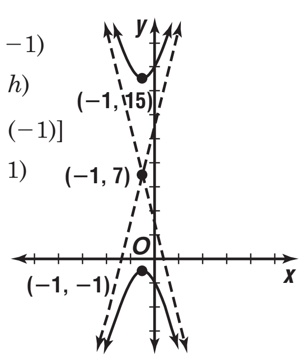
foci: $(h, k \pm c) = (-1, 7 \pm 2\sqrt{17})$

vertices: $(h, k \pm a) =$
$(-1, 7 \pm 8)$ or $(-1, 15)$, $(-1, -1)$

asymptotes: $y - k = \pm\frac{a}{b}(x - h)$

$y - 7 = \pm\frac{8}{2}[x - (-1)]$

$y - 7 = \pm 4(x + 1)$



19. $x^2 - 4y^2 + 6x - 8y = 11$

$(x^2 + 6x + ?) - 4(y^2 + 2y + ?) = 11 + ? + ?$

$(x^2 + 6x + 9) - 4(y^2 + 2y + 1) = 11 + 9 + (-4)(1)$

$(x + 3)^2 - 4(y + 1)^2 = 16$

$\frac{(x+3)^2}{16} - \frac{(y+1)^2}{4} = 1$

center: $(h, k) = (-3, -1)$

$a^2 = 16$ $\quad b^2 = 4$ $\quad c = \sqrt{a^2 + b^2}$

$a = \sqrt{16}$ or 4 $\quad b^2 = \sqrt{4}$ or 2 $\quad c = \sqrt{16 + 4}$ or $2\sqrt{5}$

transverse axis: horizontal

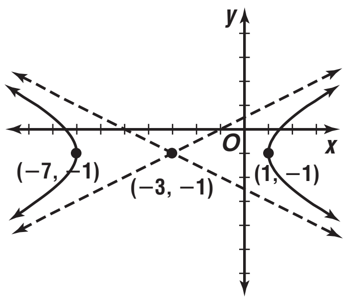
foci: $(h \pm c, k) = (-3 \pm 2\sqrt{5}, -1)$

vertices: $(h \pm a, k) = (-3 \pm 4, -1)$ or $(1, -1)$, $(-7, -1)$

asymptotes: $y - k = \pm\frac{b}{a}(x - h)$

$y - (-1) = \pm\frac{2}{4}[x - (-3)]$

$y + 1 = \pm\frac{1}{2}(x + 3)$



20. $-4x + 9y^2 - 24x - 90y + 153 = 0$

$9(y^2 - 10y + ?) - 4(x^2 + 6x + ?) = -153$

$9(y^2 - 10y + 25) - 4(x^2 + 6x + 9) = -153 + 9(25) - 4(9)$

$9(y - 5)^2 - 4(x + 3)^2 = 36$

$\frac{(y-5)^2}{4} - \frac{(x+3)^2}{9} = 1$

center: $(h, k) = (-3, 5)$

$a^2 = 4$ $\quad b^2 = 9$ $\quad c = \sqrt{a^2 + b^2}$

$a = \sqrt{4}$ or 2 $\quad b = \sqrt{9}$ or 3 $\quad c = \sqrt{4 + 9}$ or $\sqrt{13}$

transverse axis: vertical

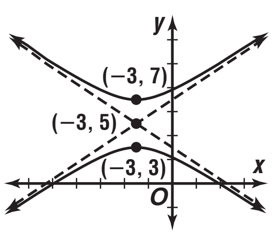
foci: $(h, k \pm c) = (-3, 5 \pm \sqrt{13})$

vertices: $(h, k \pm a) = (-3, 5 \pm 2)$ or $(-3, 7)$, $(-3, 3)$

asymptotes: $y - k = \pm\frac{a}{b}(x - h)$

$y - 5 = \pm\frac{2}{3}[x - (-3)]$

$y - 5 = \pm\frac{2}{3}(x + 3)$



21. $16y^2 - 25x^2 - 96y + 100x - 356 = 0$

$16(y^2 - 6y + ?) - 25(x^2 - 4x + ?) = 356$

$16(y^2 - 6y + 9) - 25(x^2 - 4x + 4) = 356 + 16(9) - 25(4)$

$16(y - 3)^2 - 25(x - 2)^2 = 400$

$\frac{(y-3)^2}{25} - \frac{(x-2)^2}{16} = 1$

center: $(h, k) = (2, 3)$

$a^2 = 25$ $\quad b^2 = 16$ $\quad c = \sqrt{a^2 + b^2}$

$a = \sqrt{25}$ or 5 $\quad b = \sqrt{16}$ or 4 $\quad c = \sqrt{25 + 16}$ or $\sqrt{41}$

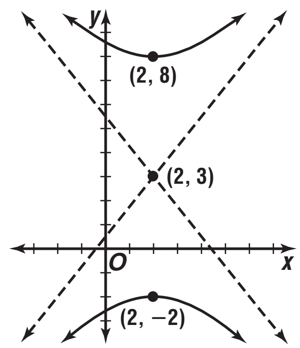
transverse axis: vertical

foci: $(h, k \pm c) = (2, 3 \pm \sqrt{41})$

vertices: $(h, k \pm a) = (2, 3 \pm 5)$ or $(2, 8)$, $(2, -2)$

asymptotes: $y - k = \pm\frac{a}{b}(x - h)$

$y - 3 = \pm\frac{5}{4}(x - 2)$



22. $36x^2 - 49y^2 - 72x - 294y = 2169$

$36(x^2 - 2x + ?) - 49(y^2 + 6y + ?) = 2169 + ? + ?$

$36(x - 2x + 1) - 49(y^2 + 6y + 9) = 2169 + 36(1) - 49(9)$

$36(x - 1)^2 - 49(y + 3)^2 = 1764$

$\frac{(x-1)^2}{49} - \frac{(y+3)^2}{36} = 1$

center: $(h, k) = (1, -3)$

$a^2 = 49$ $\quad b^2 = 36$ $\quad c = \sqrt{a^2 + b^2}$

$a = \sqrt{49}$ or 7 $\quad b = \sqrt{36}$ or 6 $\quad c = \sqrt{49 + 36}$ or $\sqrt{85}$

transverse axis: horizontal

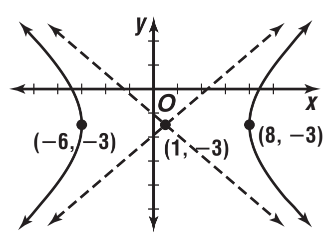
foci: $(h \pm c, k) = (1 \pm \sqrt{85}, -3)$

vertices: $(h \pm a, k) = (1 \pm 7, -3)$ or $(8, -3)$, $(-6, -3)$

asymptotes: $y - k = \pm\frac{b}{a}(x - h)$

$y - (-3) = \pm\frac{6}{7}(x - 1)$

$y + 3 = \pm\frac{6}{7}(x - 1)$



23. $25y^2 - 9x^2 - 100y - 72x - 269 = 0$

$25(y^2 - 4y + ?) - 9(x^2 + 8x + ?) = 269 + ? + ?$

$25(y^2 - 4y + 4) - 9(x^2 + 8x + 16) = 269 + 25(4) - 9(16)$

$25(y - 2)^2 - 9(x + 4)^2 = 225$

$\frac{(y-2)^2}{9} - \frac{(x+4)^2}{25} = 1$

center: $(h, k) = (-4, 2)$

$a^2 = 9$ $\quad$ $b^2 = 25$ $\quad$ $c = \sqrt{a^2 + b^2}$

$a = \sqrt{9}$ or 3 $\quad$ $b = \sqrt{25}$ or 5 $\quad$ $c = \sqrt{9 + 25}$ or $\sqrt{34}$

transverse axis: vertical

foci: $(h, k \pm c) = \left(-4, 2 \pm \sqrt{34}\right)$

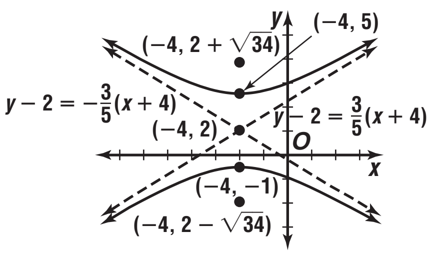
vertices: $(h, k \pm a) =$

$(-4, 2 \pm 3)$ or $(-4, 5)$, $(-4, -1)$

asymptotes: $y - k = \pm\frac{a}{b}(x - h)$

$y - 2 = \pm\frac{3}{5}[x - (-4)]$

$y - 2 = \pm\frac{3}{5}(x + 4)$



24. center: $(h, k) = (4, 3)$

transverse axis: vertical

$a = 4$, $b = 3$

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$\frac{(y-3)^2}{4^2} - \frac{(x-4)^2}{3^2} = 1$

$\frac{(y-3)^2}{16} - \frac{(x-4)^2}{9} = 1$

25. center: $(h, k) = (0, 0)$

transverse axis: horizontal

$a = 3$, $b = 3$

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-0)^2}{3^2} - \frac{(y-0)^2}{3^2} = 1$

$\frac{x^2}{9} - \frac{y^2}{9} = 1$

26. center: $(h, k) = (-4, 0)$

transverse axis: vertical

$a = 2$, $b = 1$

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$\frac{(y-0)^2}{2^2} - \frac{[x-(-4)]^2}{1^2} = 1$

$\frac{y^2}{4} - \frac{(x+4)^2}{1} = 1$

27. $c = 49$

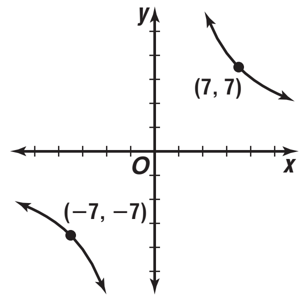
quadrants: I and III

transverse axis: $y = x$

vertices: $xy = 49$ $\quad$ $xy = 49$

$7(7) = 49$ $\quad$ $-7(-7) = 49$

$(7, 7)$ $\quad$ $(-7, -7)$



28. $c = -36$

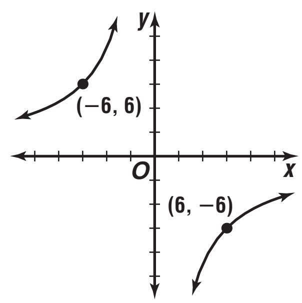
quadrants: II and IV

transverse axis: $y = -x$

vertices: $xy = -36$ $\quad$ $xy = -36$

$6(-6) = 36$ $\quad$ $-6(6) = -36$

$(6, -6)$ $\quad$ $(-6, 6)$



29. $4xy = -25$

$xy = -\frac{25}{4}$

$c = -\frac{25}{4}$

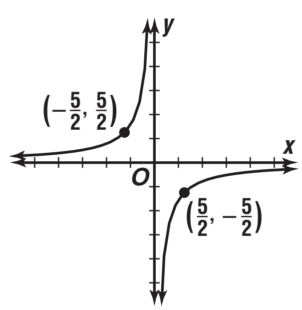
quadrants: II and IV

transverse axis: $y = -x$

vertices: $xy = -\frac{25}{4}$ $\quad$ $xy = -\frac{25}{4}$

$\frac{5}{2}\left(-\frac{5}{2}\right) = -\frac{25}{4}$ $\quad$ $-\frac{5}{2}\left(\frac{5}{2}\right) = -\frac{25}{4}$

$\left(\frac{5}{2}, -\frac{5}{2}\right)$ $\quad$ $\left(-\frac{5}{2}, \frac{5}{2}\right)$



30. $9xy = 16$

$xy = \frac{16}{9}$

$c = \frac{16}{9}$

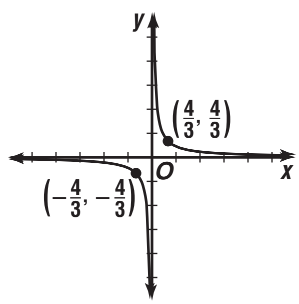
quadrants: I and III

transverse axis: $y = x$

vertices: $xy = \frac{16}{9}$ $\qquad$ $xy = \frac{16}{9}$

$\frac{4}{3}\left(\frac{4}{3}\right) = \frac{16}{9}$ $\qquad$ $-\frac{4}{3}\left(-\frac{4}{3}\right) = \frac{16}{9}$

$\left(\frac{4}{3}, \frac{4}{3}\right)$ $\qquad$ $\left(-\frac{4}{3}, -\frac{4}{3}\right)$



31. center: $(h, k) = (4, -2)$

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$\frac{[y-(-2)]^2}{2^2} - \frac{(x-4)^2}{3^2} = 1$

$\frac{(y+2)^2}{4} - \frac{(x-4)^2}{9} = 1$

32. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0+0}{2}, \frac{3+(-3)}{2}\right)$

$= (0, 0)$

transverse axis: vertical

a = distance from center to a vertex

$= |0 - 3]$ or 3

c = distance from center to a focus

$= |0 - (-9)|$ or 9

$b^2 = c^2 - a^2$

$b^2 = 9^2 - 3^2$ or 72

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$\frac{(y-0)^2}{3^2} - \frac{(x-0)^2}{72} = 1$

$\frac{y^2}{9} - \frac{x^2}{72} = 1$

33. $2a = 6$

$a = 3$

center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{5+(-5)}{2}, \frac{2+2}{2}\right)$

$= (0, 2)$

transverse axis: horizontal

c = distance from center to a focus

$= |0 - 5|$ or 5

$b^2 = c^2 - a^2$

$b^2 = 5^2 - 3^2$ or 16

$\frac{(x-b)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-0)^2}{3^2} - \frac{(y-2)^2}{16} = 1$

$\frac{x^2}{9} - \frac{(y-2)^2}{16} = 1$

34. $2b = 8$

$b = 4$

center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3+(-3)}{2}, \frac{9+(-5)}{2}\right)$

$= (-3, 2)$

transverse axis: vertical

a = distance from center to a vertex

$= |2 - 9|$ or 7

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$\frac{(y-2)^2}{7^2} - \frac{[x-(-3)]^2}{4^2} = 1$

$\frac{(y-2)^2}{49} - \frac{(x+3)^2}{16} = 1$

35. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{8+(-8)}{2}, \frac{0+0}{2}\right)$

$= (0, 0)$

transverse axis: horizontal

c = distance from center to a focus

$= |0 - 8|$ or 8

$b^2 = c^2 - a^2$ $\qquad$ $b^2 = a^2$

$a^2 = c^2 - a^2$ $\qquad$ $b^2 = 32$

$2a^2 = c^2$

$a^2 = \frac{c^2}{2}$

$a^2 = \frac{8^2}{2}$ or 32

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-0)^2}{32} - \frac{(y-0)^2}{32} = 1$

$\frac{x^2}{32} - \frac{y^2}{32} = 1$

36. centers: $(h, k) = (-3, 1)$

c = distance from center to a focus

$= |-3 - 2|$ or 5

$e = \frac{c}{a}$ $\qquad$ $b^2 = c^2 - a^2$

$\frac{5}{4} = \frac{5}{a}$ $\qquad$ $b^2 = 5^2 - 4^2$

$a = 4$ $\qquad$ $b^2 = 9$

transverse axis: horizontal

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\frac{[x-(-3)]^2}{4^2} - \frac{(y-1)^2}{9} = 1$

$\frac{(x+3)^2}{16} - \frac{(y-1)^2}{9} = 1$

37. center: $(h, k) = (4, 2)$

a = distance from center to a vertex

$= |2 - 5|$ or 3

transverse axis: vertical

$4y + 4 = 3x$

$4y + 4 - 12 = 3x - 12$

$4y - 8 = 3x - 12$

$4(y - 2) = 3(x - 4)$

$y - 2 = \frac{3}{4}(x - 4)$

$\frac{a}{b} = \frac{3}{4}$

$\frac{3}{b} = \frac{3}{4}$

$b = 4$

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$\frac{(y-2)^2}{3^2} - \frac{(x-4)^2}{4^2} = 1$

$\frac{(y-2)^2}{9} - \frac{(x-4)^2}{16} = 1$

38. center: $(h, k) = (3, -1)$
a = distance from center to a vertex
$= |3 - 5|$ or 2
transverse axis: horizontal

$$3x - 11 = 2y$$
$$3x - 11 - 4 = 2y - 4$$
$$3x - 15 = 2y - 4$$
$$3(x - 5) = 2(y - 2)$$
$$\tfrac{3}{2}(x - 5) = y - 2$$
$$y - 2 = \tfrac{3}{2}(x - 5)$$
$$\tfrac{b}{a} = \tfrac{3}{2}$$
$$\tfrac{b}{2} = \tfrac{3}{2}$$
$$b = 3$$
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-3)^2}{2^2} - \frac{[y-(-1)]^2}{3^2} = 1$$
$$\frac{(x-3)^2}{4} - \frac{(y+1)^2}{9} = 1$$

39. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0+0}{2}, \frac{8+(-8)}{2}\right) = (0, 0)$
c = distance from center to a focus
$= |0 - 8|$ or 8

$e = \frac{c}{a}$ $\qquad$ $b^2 = c^2 - a^2$
$\frac{4}{3} = \frac{8}{a}$ $\qquad$ $b^2 = 8^2 - 6^2$
$a = 6$ $\qquad$ $b^2 = 28$

transverse axis: vertical
$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
$$\frac{(y-0)^2}{6^2} - \frac{(x-0)^2}{28} = 1$$
$$\frac{y^2}{36} - \frac{x^2}{28} = 1$$

40. centers: $\left(\frac{x_1 - x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{10+(-2)}{2}, \frac{-3+(-3)}{2}\right) = (4, -3)$
c = distance from center to a focus
$= |4 - 10|$ or 6

$e = \frac{c}{a}$ $\qquad$ $b^2 = c^2 - a^2$
$\frac{6}{5} = \frac{6}{a}$ $\qquad$ $b^2 = 6^2 - 5^2$
$a = 5$ $\qquad$ $b^2 = 11$

transverse axis: horizontal
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-4)^2}{5^2} - \frac{[y-(-3)]}{11} = 1$$
$$\frac{(x-4)^2}{25} - \frac{(y+3)^2}{11} = 1$$

41. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{9+(-9)}{0}, \frac{0+0}{2}\right) = (0, 0)$
c = distance from center to a focus
$= |0 - 9|$ or 9

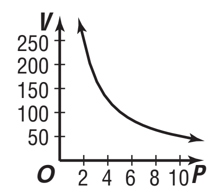
$b^2 = c^2 - a^2$ $\qquad$ $b^2 = a^2$
$a^2 = 9^2 - a^2$ $\qquad$ $b^2 = \frac{81}{2}$
$2a^2 = 81$
$a^2 = \frac{81}{2}$

transverse axis: horizontal
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-0)^2}{\frac{81}{2}} - \frac{(y-0)^2}{\frac{81}{2}} = 1$$
$$\frac{2x^2}{81} - \frac{2y^2}{81} = 1$$

42. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1+1}{2}, \frac{5+(-3)}{2}\right) = (1, 1)$
c = distance from center to a focus
$= |1 - 5|$ or 4
transverse axis: vertical

$\frac{a}{b} = \pm 2$ $\qquad$ $a^2 = (2b)^2$
$a = 2b$ $\qquad$ $a^2 = 4\left(\frac{16}{5}\right)$
$c^2 = a^2 + b^2$ $\qquad$ $a^2 = \frac{64}{5}$
$4^2 = (2b)^2 + b^2$
$16 = 5b^2$
$\frac{16}{5} = b^2$
$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
$$\frac{(y-1)^2}{\frac{64}{5}} - \frac{(x-1)^2}{\frac{16}{5}} = 1$$
$$\frac{5(y-1)^2}{64} - \frac{5(x-1)^2}{16} = 1$$

43a. quadrants: I and II
transverse axis: $y = x$



43b.
$$PV = 505$$
$$(101)V = 505$$
$$V = 5.0 \text{ dm}^3$$

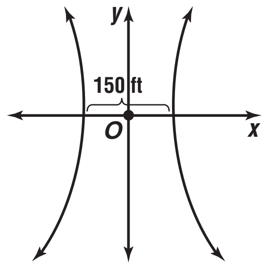
43c.
$$PV = 505$$
$$(50.5)V = 505$$
$$V = 10.0 \text{ dm}^3$$

43d. If the pressure is halved, then the volume is doubled, or $V = 2(\text{original } V)$.

44. In an equilateral hyperbola, $a = b$ and $c^2 = a^2 + b^2$.
$c^2 = a^2 + a^2$ $\qquad$ $a = b$
$c^2 = 2a^2$
$c = a\sqrt{2}$

Since $e = \frac{c}{a}$, we have
$$e = \frac{c}{a}$$
$$e = \frac{a\sqrt{2}}{a}$$
$$e = \sqrt{2}$$
Thus, the eccentricity of any equilateral hyperbola is $\sqrt{2}$.

45a.



$2a = 150$
$a = 75$

$e = \frac{c}{a}$
$\frac{5}{3} = \frac{c}{75}$
$125 = c$

$b^2 = c^2 - a^2$
$b^2 = 125^2 - 75^2$
$b^2 = 10{,}000$
$b = 100$

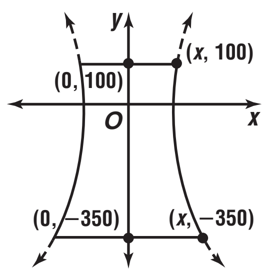
transverse axis: horizontal
center: $(h, k) = (0, 0)$

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-0)^2}{75^2} - \frac{(y-0)^2}{100^2} = 1$

$\frac{x^2}{75^2} - \frac{y^2}{100^2} = 1$

45b.



top: $\frac{x^2}{75^2} - \frac{y^2}{100^2} = 1$

$\frac{x^2}{75^2} - \frac{100^2}{100^2} = 1$ $\quad (x, y) = (x, 100)$

$\frac{x^2}{75^2} - 1 = 1$

$\frac{x^2}{75^2} = 2$

$x^2 = 11{,}250$

$x \approx 106.07$ ft

base: $\frac{x^2}{75^2} - \frac{y^2}{100^2} = 1$

$\frac{x^2}{75^2} - \frac{(-350)^2}{100^2} = 1$ $\quad (x, y) = (x, -350)$

$\frac{x^2}{75^2} - 12.25 = 1$

$\frac{x^2}{75^2} = 13.25$

$x^2 = 74{,}531.25$

$x \approx 273.00$ ft

46. The origin is located midway between stations A and B. The stations are located at the foci, so $2c = 4$ or $c = 2$ miles.

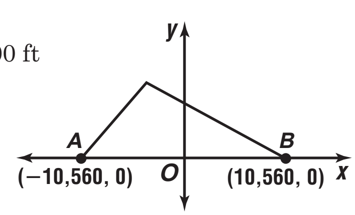
$c = 2$ mi

$c = 2 \text{ mi} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}}$

$c = 10{,}560$ ft

$d = rt$

$d = 1100(2)$ or 2200 ft



The lightning is 2200 feet farther from station B than from station A. The difference of distances equals $2a$.

$2200 = 2a$ (Definition of hyperbola)

$1100 = a$

$b^2 = c^2 - a^2$

$b = \sqrt{c^2 - a^2}$

$b = \sqrt{10{,}560^2 - 1100^2}$

$b \approx 10{,}503$

center: $(h, k) = (0, 0)$

transverse axis: horizontal

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-0)^2}{1100^2} - \frac{(y-0)^2}{10{,}503^2} = 1$

$\frac{x^2}{1100^2} - \frac{y^2}{10{,}503^2} = 1$

47. center: $(h, k) = (0, 0)$

$c = |0 - 6|$ or 6

$|PF_1 - PF_2| = 10$ $\qquad |PF_1 - PF_2| = 2a$

$2a = 10$

$a = 5$

$b^2 = c^2 - a^2$

$b^2 = 6^2 - 5^2$

$b^2 = 11$

transverse axis: horizontal

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-0)^2}{5^2} - \frac{(y-0)^2}{11} = 1$

$\frac{x^2}{25} - \frac{y^2}{11} = 1$

48a. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

center: $(h, k) = (0, 0)$

$a^2 = 16$ $\qquad b^2 = 9$

$a = \sqrt{16}$ or 4 $\qquad b = \sqrt{9}$ or 3

transverse axis: horizontal

vertices: $(h \pm a, k) = (0 \pm 4, 0)$ or $(\pm 4, 0)$

asymptotes: $y - k = \pm\frac{b}{a}(x - h)$

$y - 0 = \pm\frac{3}{4}(x - h)$

$y = \pm\frac{3}{4}x$

$\frac{y^2}{9} - \frac{x^2}{16} = 1$

center: $(h, k) = (0, 0)$

$a^2 = 9$ $\qquad b^2 = 16$

$a = \sqrt{9}$ or 3 $\qquad b = \sqrt{16}$ or 4

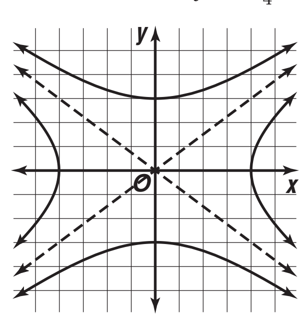
transverse axis: vertical

vertices: $(h, k \pm a) = (0, 0 \pm 3)$ or $(0, \pm 3)$

asymptotes: $y - k = \pm\frac{a}{b}(x - h)$

$y - 0 = \pm\frac{3}{4}(x - 0)$

$y = \pm\frac{3}{4}x$



48b. They are the same lines.

48c. $\frac{(y-2)^2}{25} - \frac{(x-3)^2}{16} = 1$

48d. $\frac{(x-3)^2}{16} - \frac{(y-2)^2}{25} = 1$

center: $(h, k) = (3, 2)$

$a^2 = 16$ $\quad\quad$ $b^2 = 25$

$a = \sqrt{16}$ or 4 $\quad\quad$ $b = \sqrt{25}$ or 5

transverse axis: horizontal

vertices: $(h \pm a, k) = (3 \pm 4, 2)$ or $(7, 2), (-1, 2)$

asymptotes: $y - k = \pm\frac{b}{a}(x - h)$

$y - 2 = \pm\frac{5}{4}(x - 3)$

$\frac{(y-2)^2}{25} - \frac{(x-3)^2}{16} = 1$

center: $(h, k) = (3, 2)$

$a^2 = 25$ $\quad\quad$ $b^2 = 16$

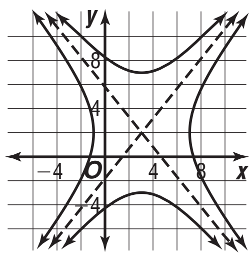
$a = \sqrt{25}$ or 5 $\quad\quad$ $b = \sqrt{16}$ or 4

transverse axis: vertical

vertices: $(h, k \pm a) = (3, 2 \pm 5)$ or $(3, 7), (3, -3)$

asymptotes: $y - k = \pm\frac{a}{b}(x - h)$

$y - 2 = \pm\frac{5}{4}(x - 3)$



49. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + 2}{2}, \frac{3 + (-3)}{2}\right)$

$= (2, 0)$

$a = 4$

c = distance from center to a focus

$= |0 - 3|$ or 3

$b^2 = a^2 - c^2$

$b^2 = 4^2 - 3^2$ or 7

major axis: vertical

$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$

$\frac{(y-0)^2}{4^2} + \frac{(x-2)^2}{7} = 1$

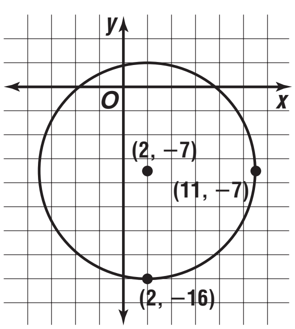
$\frac{y^2}{16} + \frac{(x-2)^2}{7} = 1$

50. $x^2 + y^2 - 4x + 14y - 28 = 0$

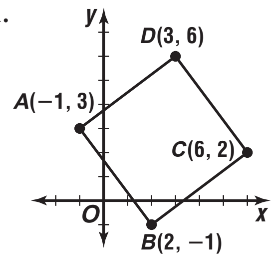
$(x^2 - 4x + ?) + (y^2 + 14y + ?) = 28 + ? + ?$

$(x^2 - 4x + 4) + (y^2 + 14y + 49) = 28 + 4 + 49$

$(x - 2)^2 + (y + 7)^2 = 81$



51.



$AB = \sqrt{(2 + 1)^2 + (-1 - 3)^2} = 5$

$BC = \sqrt{(2 - 6)^2 + (-1 - 2)^2} = 5$

$CD = \sqrt{(6 - 3)^2 + (2 - 6)^2} = 5$

$AD = \sqrt{(3 + 1)^2 + (6 - 3)^2} = 5$

Thus, $ABCD$ is a rhombus. The slope of $\overline{AD} = \frac{6 - 3}{3 + 1}$ or $\frac{3}{4}$ and the slope of $\overline{AB} = \frac{3 + 1}{-1 - 2}$ or $-\frac{4}{3}$. Thus, $\overline{AD}$ is perpendicular to $\overline{AB}$ and $ABCD$ is a square.

52. $(r, \theta) = (90, 208°)$

$(r, \theta + 360\,k°) = (90, 208° + 360(-1)°)$

$= (90, -152°)$

$(-r, \theta + (2k + 1)(180°))$

$= (-90, 208 + (2(-1) + 1)(180°))$

$= (-90, 28°)$

53. $4(-5) + -1(2) + 8(2) = -6$

No, the inner product of the two vectors is not zero.

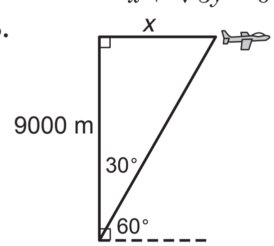
54. $x \cos \phi + y \sin \phi - p = 0$

$x \cos 60 + y \sin 60 - 3 = 0$

$\frac{1}{2}x + \frac{\sqrt{3}}{2}y - 3 = 0$

$x + \sqrt{3}y - 6 = 0$

55.



$\tan 30° = \frac{x}{9000}$

$9000 \tan 30° = x$

$5196 \approx x$

$d = rt$

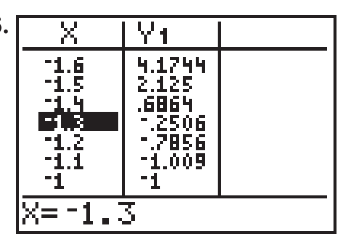
$\frac{d}{t} = r$

$\frac{5196}{15} = r$

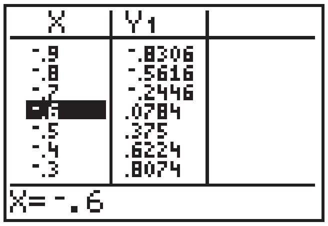
$346.4 \approx r$

about 346 m/s

56.



Since -0.2506 is closer to zero than 0.6864, the zero is about -1.3.



X	Y_1	
-.9	-.8306	
-.8	-.5616	
-.7	-.2446	
-.6	.0784	
-.5	.375	
-.4	.6224	
-.3	.8074	

X=-.6

Since 0.0784 is closer to zero than -0.2446, the zero is about -0.6.

57. Case 1: r is positive and s is negative.
Case 2: r is negative and s is positive.
I. $r^3 > s^3$ is false if r is negative.
II. $r^3 = s^2$ is false for each case.
III. $r^4 = s^4$ is true for each case.
The correct choice is C.

10-5 Parabolas

Pages 658–659 Check for Understanding

1. The equation of a parabola will have only one squared term, while the equation of a hyperbola will have two squared terms.

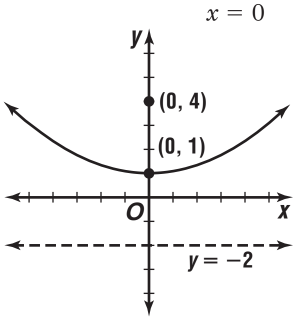
2. vertex: $(h, k) = (2, 1)$
$p = -4$
$(x - h)^2 = 4p(y - k)$
$(x - 2)^2 = 4(-4)(y - 1)$
$(x - 2)^2 = -16(y - 1)$

3. The vertex and focus both lie on the axis of symmetry. The directrix and axis of symmetry are perpendicular to each other. The focus and the point on the directrix collinear with the focus are equidistant from the vertex.

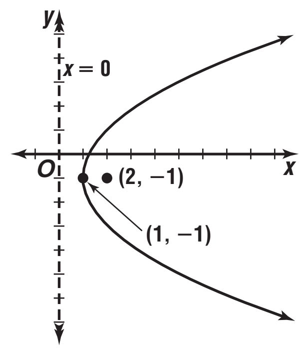
4. $(h, k) = (-4, 5)$
$p = -5$
$(y - k)^2 = 4p(x - h)$
$(y - 5)^2 = 4(-5)[x - (-4)]$
$(y - 5)^2 = -20(x + 4)$

5a. ellipse **5b.** parabola
5c. hyperbola **5d.** circle

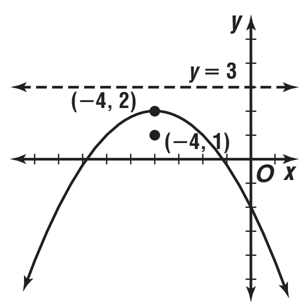
6. vertex: $(h, k) = (0, 1)$
$4p = 12$
$p = 3$
focus: $(h, k + p) = (0, 1 + 3)$ or $(0, 4)$
directrix: $y = k - p$
$y = 1 - 3$
$y = -2$
axis of symmetry: $x = h$
$x = 0$



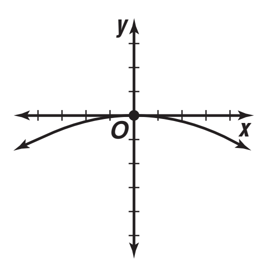
7. $y^2 - 4x + 2y + 5 = 0$
$y^2 + 2y = 4x - 5$
$y^2 + 2y + ? = 4x - 5 + ?$
$y^2 + 2y + 1 = 4x - 5 + 1$
$(y + 1) = 4(x - 1)$
vertex: $(h, k) = (1, -1)$
$4p = 4$
$p = 1$
focus: $(h + p, k) = (1 + 1, -1)$ or $(2, -1)$
directrix: $x = h - p$
$x = 1 - 1$
$x = 0$
axis of symmetry: $y = k$
$y = -1$



8. $x^2 + 8x + 4y + 8 = 0$
$x^2 + 8x = -4y - 8$
$x^2 + 8x + ? = -4y - 8 + ?$
$x^2 + 8x + 16 = -4y - 8 + 16$
$(x + 4)^2 = -4(y - 2)$
vertex: $(h, k) = (-4, 2)$
$4p = -4$
$p = -1$
focus: $(h, k + p) = \big(-4, 2 + (-1)\big)$ or $(-4, 1)$
directrix: $y = k - p$
$y = 2 - (-1)$
$y = 3$
axis of symmetry: $x = h$
$x = -4$



9. vertex: $(h, k) = (0, 0)$
opening: downward
$p = -4$
$(x - h)^2 = 4p(y - k)$
$(x - 0)^2 = 4(-4)(y - 0)$
$x^2 = -16y$



10. $(y - k)^2 = 4p(x - h)$

$(-1 - 5)^2 = 4p[2 - (-7)]$ *$(h, k) = (-7, 5)$;*

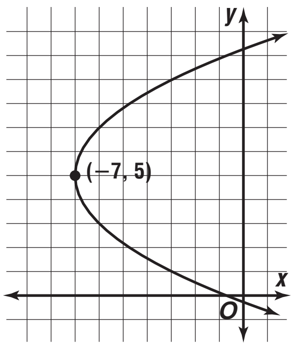
$(-6)^2 = 36p$ *$(x, y) = (2, -1)$*

$1 = p$

$(y - k)^2 = 4p(x - h)$

$(y - 5)^2 = 4(1)[x - (-7)]$

$(y - 5)^2 = 4(x + 7)$



11. vertex: $(h, k) = (4, -3)$

opening: upward

$(x - h)^2 = 4p(y - k)$

$(5 - 4)^2 = 4p[2 - (-3)]$ *$(h, k) = (4, -3)$;*

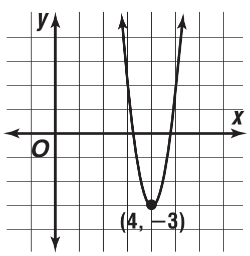
$1^2 = 20p$ *$(x, y) = (5, 2)$*

$\frac{1}{20} = p$

$(x - h)^2 = 4p(y - k)$

$(x - 4)^2 = 4\left(\frac{1}{20}\right)(y + 3)$

$(x - 4)^2 = \frac{1}{5}(y + 3)$

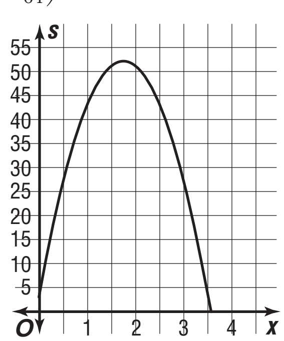


12a.

$$s = v_0t - 16t^2 + 3$$
$$s = 56t - 16t^2 + 3$$
$$-16t^2 + 56t = s - 3$$
$$-16\left(t^2 - \frac{7}{5}t + ?\right) = s - 3 + ?$$
$$-16\left(t^2 - \frac{7}{2}t + \frac{49}{16}\right) = s - 3 + (-16)\left(\frac{49}{16}\right)$$
$$-16\left(t - \frac{7}{4}\right)^2 = s - 52$$
$$\left(t - \frac{7}{4}\right)^2 = -\frac{1}{16}(s - 52)$$
$$\left(t - \frac{7}{4}\right)^2 = 4\left(-\frac{1}{64}\right)(s - 52)$$

vertex: $(h, k) = \left(\frac{7}{4}, 52\right)$

opening: downward



12b. The maximum height is $s = 52$ ft.

12c. Let $s = 0$.

$s = 56t - 16t^2 + 3$

$0 = -16t^2 + 56t + 3$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-56 \pm \sqrt{56^2 - 4(-16)(3)}}{2(-16)}$

$t \approx 3.6$ or -0.05

3.6 s

Pages 659–661 Exercises

13. vertex: $(h, k) = (0, 0)$

$4p = 8$

$p = 2$

focus: $(h + p, k) = (0 + 2, 0)$ or $(2, 0)$

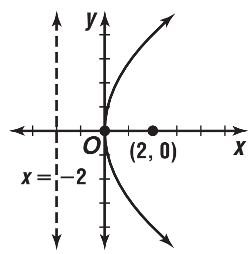
directrix: $x = h - p$

$x = 0 - 2$

$x = -2$

axis of symmetry: $y = k$

$y = 0$



14. vertex: $(h, k) = (0, 3)$

$4p = -4$

$p = 1$

focus: $(h, k + p) = (0, 3 + (-1))$ or $(0, 2)$

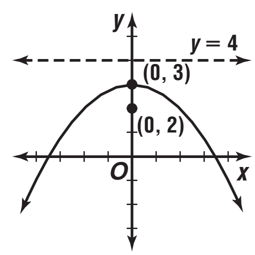
directrix: $y = k - p$

$y = 3 - (-1)$

$y = 4$

axis of symmetry: $x = h$

$x = 0$



15. vertex: $(h, k) = (0, 6)$

$4p = 4$

$p = 1$

focus: $(h + p, k) = (0 + 1, 6)$ or $(1, 6)$

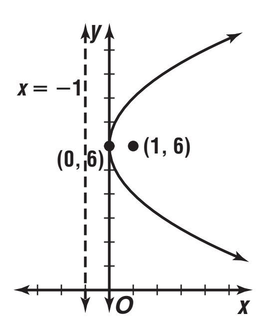
directrix: $x = h - p$

$x = 0 - 1$

$x = -1$

axis of symmetry: $y = k$

$y = 6$



16. $y^2 + 12x = 2y - 13$

$y^2 - 2y = -12x - 13$

$y^2 - 2y + ? = -12x - 13 + ?$

$y^2 - 2y + 1 = -12x - 13 + 1$

$(y - 1)^2 = -12(x + 1)$

vertex: $(h, k) = (-1, 1)$

$4p = -12$

$p = -3$

focus: $(h + p, k) = (-1 + (-3), 1)$ or $(-4, 1)$

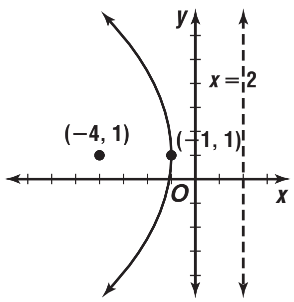
directrix: $x = h - p$

$x = -1 - (-3)$

$x = 2$

axis of symmetry: $y = k$

$y = 1$



17. $y - 2 = x^2 - 4x$

$x^2 - 4x = y - 2$

$x^2 - 4x + ? = y - 2 + ?$

$x^2 - 4x + 4 = y - 2 + 4$

$(x - 2)^2 = y + 2$

vertex: $(h, k) = (2, -2)$

$4p = 1$

$p = \frac{1}{4}$

focus: $(h, k + p) = \left(2, -2 + \frac{1}{4}\right)$ or $\left(2, -\frac{7}{4}\right)$

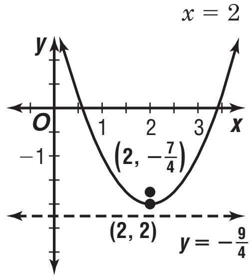
directrix: $y = k - p$

$y = -2 - \frac{1}{4}$

$y = -\frac{9}{4}$

axis of symmetry: $x = h$

$x = 2$



18. $x^2 + 10x + 25 = -8y + 24$

$(x + 5)^2 = -8(y - 3)$

vertex: $(h, k) = (-5, 3)$

$4p = -8$

$p = -2$

focus: $(h, k + p) = (-5, 3 + (-2))$ or $(-5, 1)$

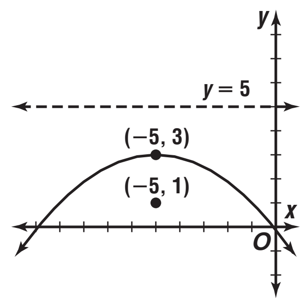
directrix: $y = k - p$

$y = 3 - (-2)$

$y = 5$

axis of symmetry: $x = h$

$x = -5$



19. $y^2 - 2x + 14y = -41$

$y^2 + 14y = 2x - 41$

$y^2 + 14y + ? = 2x - 41 + ?$

$y^2 + 14y + 49 = 2x - 41 + 49$

$(y + 7)^2 = (2x + 4)$

vertex: $(h, k) = (-4, -7)$

$4p = 2$

$p = \frac{2}{4}$ or $\frac{1}{2}$

focus: $(h + p, k) = \left(-4 + \frac{1}{2}, -7\right)$ or $\left(-\frac{7}{2}, -7\right)$

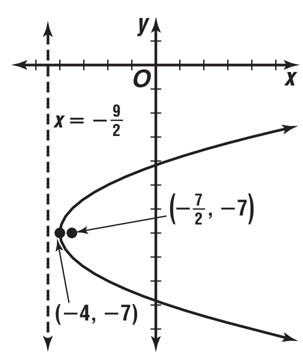
directrix: $x = h - p$

$x = -4 - \frac{1}{2}$

$x = -\frac{9}{2}$

axis of symmetry: $y = k$

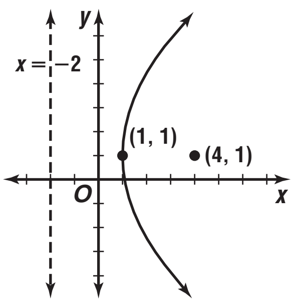
$y = -7$



20. $y^2 - 2y - 12x + 13 = 0$

$$y^2 - 2y = 12x - 13$$
$$y^2 - 2y + ? = 12x - 13 + ?$$
$$y^2 - 2y + 1 = 12x - 13 + 1$$
$$(y - 1)^2 = 12(x - 1)$$

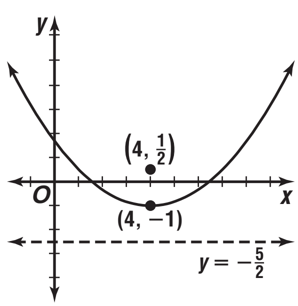
vertex: $(h, k) = (1, 1)$
$4p = 12$
$p = 3$
focus: $(h + p, k) = (1 + 3, 1)$ or $(4, 1)$
directrix: $x = h - p$
$x = 1 - 3$
$x = -2$
axis of symmetry: $y = k$
$y = 1$



21. $2x^2 - 12y - 16x + 20 = 0$

$$2x^2 - 16x = 12y - 20$$
$$2(x^2 - 8x + ?) = 12y - 20 + ?$$
$$2(x^2 - 8x + 16) = 12y - 20 + 2(16)$$
$$2(x - 4)^2 = 12y + 12$$
$$(x - 4)^2 = 6(y + 1)$$

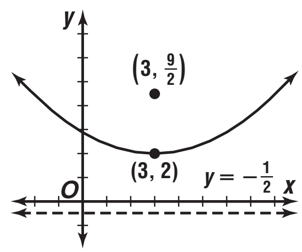
vertex: $(h, k) = (4, -1)$
$4p = 6$
$p = \frac{6}{4}$ or $\frac{3}{2}$
focus: $(h, k + p) = \left(4, -1 + \frac{3}{2}\right)$ or $\left(4, \frac{1}{2}\right)$
directrix: $y = k - p$
$y = -1 - \frac{3}{2}$
$y = -\frac{5}{2}$
axis of symmetry: $x = h$
$x = 4$



22. $3x^2 - 30x - 18x + 87 = 0$

$$3x^2 - 18x = 30y - 87$$
$$3(x^2 - 6x + ?) = 30y - 87 + ?$$
$$3(x^2 - 6x + 9) = 30y - 87 + 3(9)$$
$$3(x - 3)^2 = 30y - 60$$
$$(x - 3)^2 = 10y - 20$$
$$(x - 3)^2 = 10(y - 2)$$

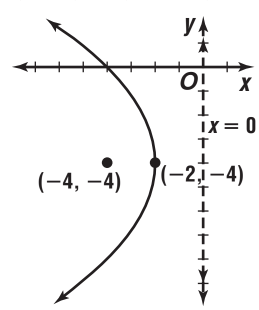
vertex: $(h, k) = (3, 2)$
$4p = 10$
$p = \frac{10}{4}$ or $\frac{5}{2}$
focus: $(h, k + p) = \left(3, 2 + \frac{5}{2}\right)$ or $\left(3, \frac{9}{2}\right)$
directrix: $y = k - p$
$y = 2 - \frac{5}{2}$
$y = -\frac{1}{2}$
axis of symmetry: $x = h$
$x = 3$



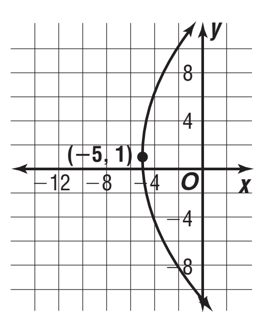
23. $2y^2 + 16y + 16x + 64 = 0$

$$2y^2 + 16y = -16x - 64$$
$$2(y^2 + 8y + ?) = -16x - 64 + ?$$
$$2(y^2 + 8y + 16) = -16x - 64 + 2(16)$$
$$2(y + 4)^2 = -16x - 32$$
$$(y + 4)^2 = -8x - 16$$
$$(y + 4)^2 = -8(x + 2)$$

vertex: $(h, k) = (-2, -4)$
$4p = -8$
$p = -2$
focus: $(h + p, k) = (-2 + (-2), -4)$ or $(-4, -4)$
directrix: $x = h - p$
$x = -2 - (-2)$
$x = 0$
axis of symmetry: $y = k$
$y = -4$



24. vertex: $(h, k) = (-5, 1)$
opening: right
$h + p = 2$
$-5 + p = 2$
$p = 7$
$(y - k)^2 = 4p(x - h)$
$(y - 1)^2 = 4(7)[x - (-5)]$
$(y - 1)^2 = 28(x + 5)$



25. opening: left

focus: $(h + p, k) = (0, 6)$

$h + p = 0$ $\qquad k = 6$

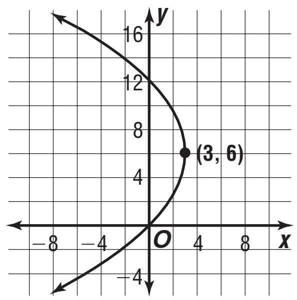
$h + (-3) = 0$

$h = 3$

$(y - k)^2 = 4p(x - h)$

$(y - 6)^2 = 4(-3)(x - 3)$

$(y - 6)^2 = -12(x - 3)$



26. opening: upward

vertex: $(h, k) = \left(4, \frac{5-1}{2}\right)$ or $(4, -3)$

focus: $(h, k + p) = (4, -1)$

$k + p = -1$

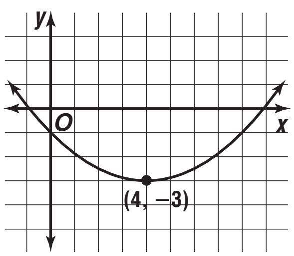
$-3 + p = -1$

$p = 2$

$(x - b)^2 = 4p(y - k)$

$(x - 4)^2 = 4(2)[y - (-3)]$

$(x - 4)^2 = 8(y + 3)$



27. opening: downward

vertex: $(h, k) = (4, 3)$

$(x - h)^2 = 4p(y - k)$

$(5 - 4)^2 = 4p(2 - 3)$ $\qquad$ *(h, k) = (4, 3);*

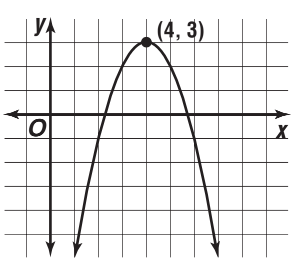
$1^2 = -4p$ $\qquad$ *(x, y) = (5, 2)*

$-\frac{1}{4} = p$

$(x - h)^2 = 4p(y - k)$

$(x - 4)^2 = 4\left(-\frac{1}{4}\right)(y - 3)$

$(x - 4)^2 = -(y - 3)$



28. vertex: $(h, k) = (-2, -3)$

$(y - k)^2 = 4p(x - h)$

$[1 - (-3)]^2 = 4p[-3 - (-2)]$ $\qquad$ *(h, k) = (−2, −3);*

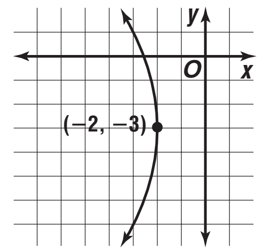
$4^2 = 4p$ $\qquad$ *(x, y) = (−3, 1)*

$-4 = p$

$(y - k)^2 = 4p(x - h)$

$(y + 3)^2 = 4(-4)(x + 2)$

$(y + 3)^2 = -16(x + 2)$



29. opening: upward

$p = 2$

focus: $(h, k + p) = (-1, 7)$

$h = -1$ $\qquad k + p = 7$

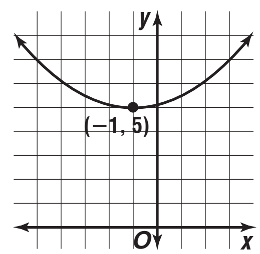
$k + 2 = 7$

$k = 5$

$(x - h)^2 = 4p(y - k)$

$[x - (-1)]^2 = 4(2)(y - 5)$

$(x + 1)^2 = 8(y - 5)$



30. opening: downward

vertex: $(h, k) = (5, -3)$ $\qquad$ (maximum)

$(x - h)^2 = 4p(y - k)$

$(1 - 5)^2 = 4p[-7 - (-3)]$ $\qquad$ *(h, k) = (5, −3);*

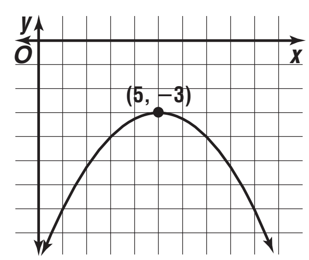
$(-4)^2 = -16p$ $\qquad$ *(x, y) = (1, −7)*

$-1 = p$

$(x - h)^2 = 4p(y - k)$

$(x - 5)^2 = 4(-1)(y + 3)$

$(x - 5)^2 = -4(y + 3)$



31. opening: right

vertex: $(h, k) = (-1, 2)$

$(y - k)^2 = 4p(x - h)$

$(0 - 2)^2 = 4p[0 - (-1)]$ $\qquad$ *(h, k) = (−1, 2);*

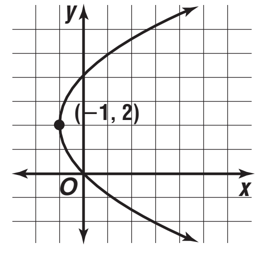
$(-2)^2 = 4p$ $\qquad$ *(x, y) = (0, 0)*

$1 = p$

$(y - k)^2 = 4p(x - h)$

$(y - 2)^2 = 4(1)[x - (-1)]$

$(y - 2)^2 = 4(x - 1)$



32. opening: upward

$h = \frac{x_1 + x_2}{2}$

$= \frac{1+2}{2}$ or $\frac{3}{2}$

vertex: $(h, k) = \left(\frac{3}{2}, 0\right)$

$(x - h)^2 = 4p(y - k)$

$\left(1 - \frac{3}{2}\right)^2 = 4p(1 - 0)$ $\quad$ *$(h, k) = \left(\frac{3}{2}, 0\right)$;*

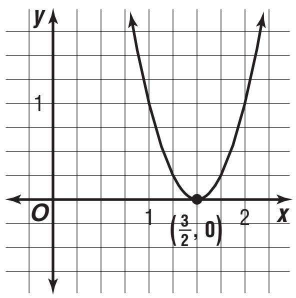
$\frac{1}{4} = 4p$ $\quad$ *$(x, y) = (1, 1)$*

$\frac{1}{16} = p$

$(x - h)^2 = 4p(y - k)$

$\left(x - \frac{3}{2}\right)^2 = 4\left(\frac{1}{16}\right)(y - 0)$

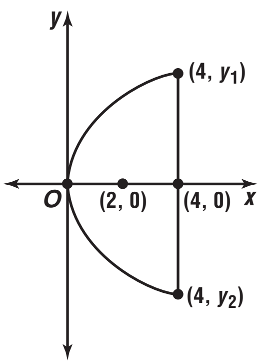
$\left(y - \frac{3}{2}\right) = \frac{1}{4}y$



33a. vertex: $(h, k) = (0, 0)$

depth: $x = 4$

$p = 2$



$(y - k)^2 = 4p(x - h)$

$(y - 0)^2 = 4(2)(x - 0)$ $\quad$ *$(h, k) = (0, 0)$;*

$y^2 = 8x$ $\quad$ *$p = 2$*

$y = \pm\sqrt{8x}$

$y = \pm\sqrt{8(4)}$

$y = \pm 4\sqrt{2}$

$y_1 = 4\sqrt{2}, y_2 = -4\sqrt{2}$

diameter $= |y_1 - y_2|$

$= |4\sqrt{2} - (-4\sqrt{2})|$

$= 8\sqrt{2}$ in.

33b. depth: $x = 1.25(4)$

$x = 5$

$(y - k)^2 = 4p(x - h)$

$(y - 0)^2 = 4(2)(x - 0)$ $\quad$ *$(h, k) = (0, 0)$;*

$y^2 = 8x$ $\quad$ *$p = 2$*

$y = \pm\sqrt{8x}$

$y = \pm\sqrt{8(5)}$

$y = \pm 2\sqrt{10}$

$y_1 = 2\sqrt{10}, y_2 = -2\sqrt{10}$

diameter $= |y_1 - y_2|$

$= |2\sqrt{10} - (-2\sqrt{10})|$

$= 4\sqrt{10}$ in.

34a. Let y = income per flight.

Let x = the number of \$10 price decreases.

Income = number of passengers · cost of a ticket

$y = (110 + 20x) \cdot (140 - 10x)$

$y =$

$15{,}400 - 1100x + 2800x - 200x^2$

$y = -200\left(x^2 - \frac{17}{2}x\right) + 15{,}400$

$y - 15{,}400 = -200\left(x^2 - \frac{17}{2}x\right)$

$y - 15{,}400 - 200\left(\frac{17}{4}\right)^2 = -200\left[x^2 - \frac{17}{2}x + \left(\frac{17}{9}\right)^2\right]$

$-\frac{1}{200}(y - 19{,}012.5) = \left(x - \frac{17}{4}\right)^2$

The vertex of the parabola is at $\left(\frac{17}{4}, 19{,}012.5\right)$, and because p is negative it opens downward. So the vertex is a maximum and the number of \$10 price decreases is $\frac{17}{4}$ or 4.25.

number of passengers $= 110 + 20x$

$= 110 + 20(4.25)$

$= 195$

However, the flight can transport only 180 people.

number of passengers $= 110 + 20x$

$180 = 110 + 20x$

$3.5 = x$

Therefore, there should be 3.5 \$10 price decreases.

cost of ticket $= 140 - 10x$

$= 140 - 10(3.5)$

$= \$105$

34b. Let y = income per flight.

Let x = the number of \$10 price decreases.

Income = number of passengers · cost of a ticket

$y = (110 + 10x) \cdot (140 - 10x)$

$y = 15{,}400 + 300x - 100x^2$

$y = -100(x^2 - 3x) + 15{,}400$

$y - 15{,}400 = -100(x^2 - 3x)$

$y - 15{,}400 - 100\left(\frac{3}{2}\right)^2 = -100\left[x^2 - 3x + \left(\frac{3}{2}\right)^2\right]$

$y - 15{,}625 = -100\left(x - \frac{3}{2}\right)^2$

$-\frac{1}{100}(y - 15{,}625) = \left(x - \frac{3}{2}\right)^2$

The vertex of the parabola is at $\left(\frac{3}{2}, 15{,}625\right)$, and because p is negative, it opens downward. So the vertex is a maximum and the number of \$10 price decreases is $\frac{3}{2}$ or 1.5.

number of passengers $= 110 + 10x$

$= 110 + 10(1.5)$

$= 125$

This is less than 180, so the new ticket price can be found using 1.5 \$10 price decreases.

cost of a ticket $= 140 - 10x$

$= 140 - 10(1.5)$

$= \$125$

35a. Let $(h, k) = (0, 0)$.

$x^2 = 4py$

$x^2 = 4\left(\frac{1}{8}\right)y \qquad p = \frac{1}{8}$

$2x^2 = y$

$x^2 = 4py$

$x^2 = 4\left(\frac{1}{4}\right)y \qquad p = \frac{1}{4}$

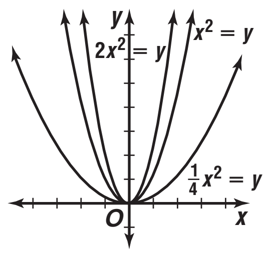
$x^2 = y$

$x^2 = 4py$

$x^2 = -1(1)y \qquad p = 1$

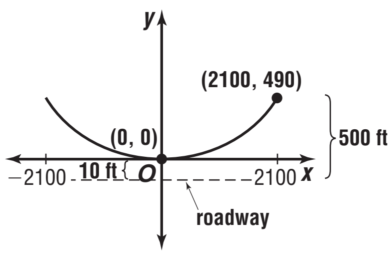
$\frac{1}{4}x = y$

The opening becomes narrower.



35b. The opening becomes wider.

36a. Sample answer:
opening: upward
vertex: $(h, k) = (0, 0)$



$(x - h)^2 = 4p(y - k)$

$(2100 - 0)^2 = 4p(490 - 0) \qquad (h, k) = (0, 0)$

$2250 = p \qquad (x, y) = (2100, 490)$

$(x - h)^2 = 4p(y - k)$

$(x - 0)^2 = 4(2250)(y - 0)$

$x^2 = 9000y$

36b. $x^2 = 9000y$

$(720)^2 = 9000y$

$57.6 = y$

$57.6 + 10 = 67.6$ ft

37.

$(y - k)^2 = 4p(x - h)$

$y^2 - 2ky + k^2 = 4px - 4ph$

$y^2 - 4px - 2ky + k^2 + 4ph = 0$

$y^2 + Dx + Ey + F = 0$

$(x - h)^2 = 4p(y - k)$

$x^2 - 2hx + h^2 = 4py - 4pk$

$x^2 - 4py - 2hx + h^2 + 4pk = 0$

$x^2 + Dx + Ey + F = 0$

38a. $4p = 8$

$p = 2$ or $p = -2$

opening: right

$(y - k)^2 = 4p(x - h)$

$(y - 2)^2 = 4(2)[x - (-3)]$

$(y - 2)^2 = 8(x + 3)$

opening: left

$(y - k)^2 = 4p(x - h)$

$(y - 2)^2 = 4(-2)[x - (-3)]$

$(y - 2)^2 = -8(x + 3)$

38b. $4p = -16$

$p = -4$

focus of parabola = center of circle

vertex: $(h, k) = (1, 4)$

focus: $(h, k + p) = (1, 4 + (-4))$ or $(1, 0)$

diameter = latus rectum
= 16

radius $= \frac{1}{2}(16)$
$= 8$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 1)^2 + (y - 0)^2 = 8^2$

$(x - 1)^2 + y^2 = 64$

39. center: $(h, k) = (2, 3)$

$a^2 = 25 \qquad b^2 = 16$

$a = \sqrt{25}$ or 5 $\qquad b = \sqrt{16}$ or 4

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{25 + 16}$ or $\sqrt{41}$

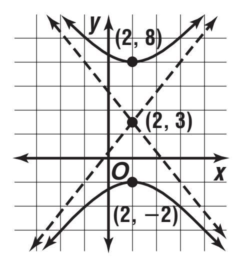
transverse axis: vertical

foci: $(h, k \pm c) = (2, 3 \pm \sqrt{41})$

vertices: $(h, k \pm a) = (2, 3 \pm 5)$ or $(2, 8), (2, -2)$

asymptotes: $y - k = \pm\frac{a}{b}(x - h)$

$y - 3 = \pm\frac{5}{4}(x - 2)$



40. $4x^2 + 25y^2 + 250y + 525 = 0$

$4x^2 + 25(y^2 + 10y + ?) = -525 + ?$

$4x^2 + 25(y^2 + 10y + 25) = -525 + 25(25)$

$4x^2 + 25(y + 5)^2 = 100$

$\frac{x^2}{25} + \frac{(y + 5)^2}{4} = 1$

center: $(h, k) = (0, -5)$

$a^2 = 25 \qquad b^2 = 4$

$a = 25$ or 5 $\qquad b = \sqrt{4}$ or 2

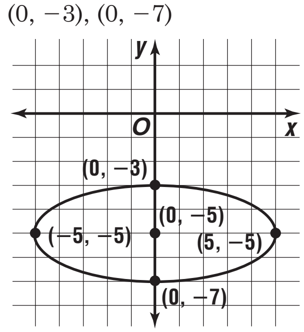
$c = \sqrt{a^2 - b^2}$

$c = \sqrt{25 - 4}$ or $\sqrt{21}$

foci: $(h \pm c, k) = (0 \pm \sqrt{21}, -5)$ or $(\pm\sqrt{21}, -5)$

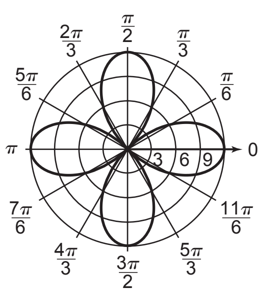
major axis vertices: $(h \pm a, k) = (0 \pm 5, -5)$ or $(\pm 5, -5)$

minor axis vertices: $(h, k \pm b) = (0, -5 \pm 2)$ or $(0, -3), (0, -7)$



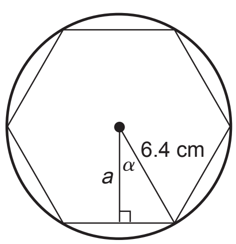
41.

θ	$12 \cos 2\theta$	(r, θ)
0	12	(12, 0)
$\frac{\pi}{6}$	6	$\left(6, \frac{\pi}{6}\right)$
$\frac{\pi}{3}$	−6	$\left(-6, \frac{\pi}{3}\right)$
$\frac{\pi}{2}$	−12	$\left(-12, \frac{\pi}{2}\right)$
$\frac{2\pi}{3}$	−6	$\left(-6, \frac{2\pi}{3}\right)$
$\frac{5\pi}{6}$	6	$\left(6, \frac{5\pi}{6}\right)$
π	12	$(12, \pi)$
$\frac{7\pi}{6}$	6	$\left(6, \frac{7\pi}{6}\right)$
$\frac{4\pi}{3}$	−6	$\left(-6, \frac{4\pi}{3}\right)$
$\frac{3\pi}{2}$	−12	$\left(-12, \frac{3\pi}{2}\right)$
$\frac{5\pi}{3}$	−6	$\left(-6, \frac{5\pi}{3}\right)$
$\frac{11\pi}{6}$	6	$\left(6, \frac{11\pi}{6}\right)$



42. $2\pi n$, where n is any integer

43.



The measure of α is $360° \div 12$ or $30°$.

$$\cos 30° = \frac{a}{6.4}$$
$$6.4 \cos 30° = a$$
$$5.5 \approx a; 5.5 \text{ cm}$$

44.

$g(x) = \frac{4}{x^2 + 1}$	
x	$y = g(x)$
−10,000	4×10^{-8}
−1000	4×10^{-6}
−100	4×10^{-4}
−10	0.04
0	4
10	0.04
100	4×10^{-4}
1000	4×10^{-6}
10,000	4×10^{-8}

$y \to 0$ as $x \to \infty$, $y \to 0$ as $x \to -\infty$

45. $19 < t < 14 + 19$

$19 < t < 33$

$t = 3^3$ or 27

$$\begin{aligned}\text{perimeter} &= 14 + 19 + t \\ &= 14 + 19 + 27 \\ &= 60\end{aligned}$$

The correct choice is C.

Page 661 Mid-Chapter Quiz

1a. $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6 - 3)^2 + (9 - 3)^2}$

$= \sqrt{3^2 + 6^2}$

$= \sqrt{45}$

$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(9 - 6)^2 + (3 - 9)^2}$

$= \sqrt{3^2 + (-6)^2}$

$= \sqrt{45}$

Since $AB = BC$, triangle ABC is isosceles.

1b. $AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(9 - 3)^2 + (3 - 3)^2}$

$= \sqrt{6^2 + 0^2}$

$= 6$

perimeter $= AB + BC + AC$

$= \sqrt{45} + \sqrt{45} + 6$

≈ 19.42 units

2. Diagonals of a rectangle intersect at their midpoint.

midpoint of $\overline{AC} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-4 + 5}{2}, \frac{9 + 5}{2}\right)$

$= (0.5, 7)$

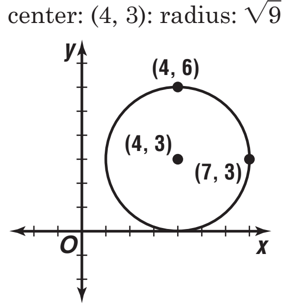
3. $x^2 + y^2 - 6y - 8x = -16$

$(x^2 - 8x + ?) + (y^2 - 6y + ?) = -16 + ? + ?$

$(x^2 - 8x + 16) + (y^2 - 6y + 9) = -16 + 16 + 9$

$(x - 4)^2 + (y - 3)^2 = 9$

center: (4, 3): radius: $\sqrt{9}$ or 3

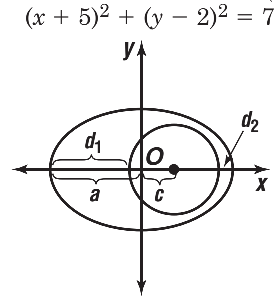


4. $(x - h)^2 + (y - k)^2 = r^2$

$[x - (-5)]^2 + (y - 2)^2 = \left(\sqrt{7}\right)^2$

$(x + 5)^2 + (y - 2)^2 = 7$

5a.

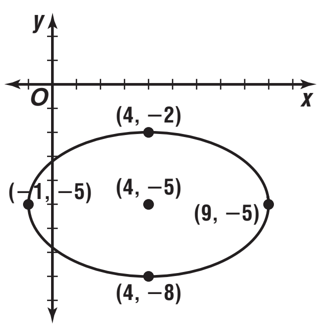


Let d_1 be the greatest distance from the satellite to Earth. Let d_2 be the least distance from the satellite to Earth.

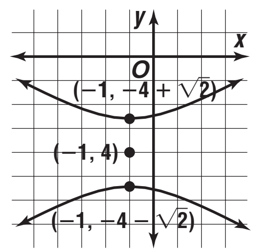
$a = \frac{1}{2}(10{,}440)$
$a = 5220$
$\frac{c}{a} = e$
$\frac{c}{5220} = 0.16$
$c = 835.20$
radius of Earth $= \frac{1}{2}(7920)$
$= 3960$
$d_1 = a + c -$ Earth radius
$d_1 = 5220 + 835.20 - 3960$
$d_1 = 2095.2$ miles
$d_2 =$ major axis $- d_1 -$ Earth diameter
$d_2 = 10{,}440 - 2095.2 - 7920$
$d_2 = 424.8$ miles

5b. $(h, k) = (0,0)$
$a = 5220$
$b^2 = a^2(1 - e^2)$
$b^2 = (5220)^2\,(1 - 0.16^2)$
$b^2 = 26{,}550{,}840.96$
$$\frac{(x-b)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-0)^2}{5220} + \frac{(y-0)^2}{26{,}550{,}840.96} = 1$$
$$\frac{x^2}{27{,}248{,}400} + \frac{y^2}{26{,}550{,}840.96} = 1$$

6. $9x^2 + 25y^2 - 72x + 250y + 554 = 0$
$9(x^2 - 8x + ?) + 25(y^2 + 10y + ?) =$
$-544 + ? + ?$
$9(x^2 - 8x + 16) + 25(y^2 + 10y + 25) =$
$-544 + 9(16) + 25(25)$
$9(x-4)^2 + 25(y+5)^2 = 225$
$$\frac{(x-4)^2}{25} + \frac{(y+5)^2}{9} = 1$$
center: $(h, k) = (4, -5)$
$a^2 = 25$ $\quad b^2 = 9$ $\quad c = \sqrt{a^2 - b^2}$
$a = 5$ $\quad b = 3$ $\quad c = \sqrt{25 - 9}$ or 4
major axis vertices: $(h \pm a, k) = (4 \pm 5, -5)$ or $(9, -5), (-1, -5)$
minor axis vertices: $(h, k \pm b) = (4, -5 \pm 3)$ or $(4, -2), (4, -8)$
foci: $(h \pm c, k) = (4 \pm 4, -5)$ or $(8, -5), (0, -5)$

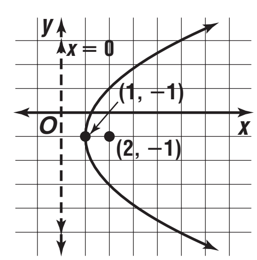


7. $3y^2 + 24y - x^2 - 2x + 41 = 0$
$3(y^2 + 8y + ?) - (x^2 + 2x + ?) = -41 + ? + ?$
$3(y^2 + 8y + 16) - (x^2 + 2x - 1) = -41 + 3(16) - 1$
$3(y+4)^2 - (x+1)^2 = 6$
$$\frac{(y+4)^2}{2} - \frac{(x+1)^2}{6} = 1$$
center: $(h, k) = (-1, -4)$
$a^2 = 2$ $\quad b^2 = 6$ $\quad c = \sqrt{a^2 - b^2}$
$a = \sqrt{2}$ $\quad b = \sqrt{6}$ $\quad c = \sqrt{2 + 6}$ or $2\sqrt{2}$
transverse axis: vertical
vertices: $(h, k \pm a) = \left(-1, -4 \pm \sqrt{2}\right)$
foci: $(h, k \pm c) = \left(-1, -4 \pm 2\sqrt{2}\right)$
asymptotes: $y - k = \pm\frac{a}{b}(x - h)$
$$y - (-4) = \pm\frac{\sqrt{2}}{\sqrt{6}}\,[x - (-1)]$$
$$y + 4 = \pm\frac{\sqrt{3}}{3}\,(x + 1)$$



8. To find the center, find the intersection of the asymptotes.
$y = -2x + 4$
$2x = -2x + 4$
$4x = 4$
$x = 1$
$y = 2 \cdot 1$
$y = 2$
The center is at (1, 2).
Notice that (4, 2) must be a vertex and a equals $4 - 1$ or 3.
Point A has an x-coordinate of 4.
Since $y = 2x$, the y-coordinate is $2 \cdot 4$ or 8.
The value of b is $8 - 2$ or 6.
The equation is $\frac{(x-1)^2}{9} - \frac{(y-2)^2}{36} = 1$.

9. $y^2 - 4x + 2y + 5 = 0$
$y^2 + 2y = 4x - 5$
$y^2 + 2y + ? = 4x - 5 + ?$
$y^2 + 2y + 1 = 4x - 5 + 1$
$(y+1)^2 = 4(x-1)$
vertex: $(h, k) = (1, -1)$
$4p = 4$
$p = 1$
focus: $(h + p, k) = (1 + 1, -1)$ or $(2, -1)$
axis of symmetry: $y = k$
$y = -1$
directrix: $x = h - p$
$x = 1 - 1$
$x = 0$



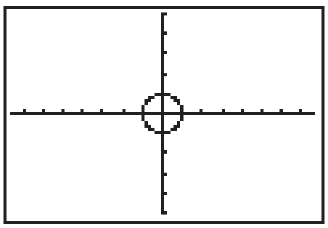
10. vertex: $(h, k) = (5, -1)$

$(x - h)^2 = 4p(y - k)$ $\quad$ $(h, k) = (5, -1)$
$(9 - 5)^2 = 4p[-2 - (-1)]$ $\quad$ $(x, y) = (9, -2)$
$4^2 = -4p$
$-4 = p$
$(x - h)^2 = 4p(y - k)$
$(x - 5)^2 = 4(-4)[y - (-1)]$
$(x - 5)^2 = -16(y + 1)$

10-6 Rectangular and Parametric Forms of Conic Sections

Page 665 Graphing Calculator Exploration

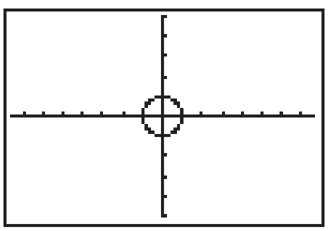
1.



Tmin: [0, 6.28] step: 0.1
$[-7.58, 7.58]$ scl: 1 by $[-5, 5]$ scl: 1

1a. $(-1, 0)$ **1b.** clockwise

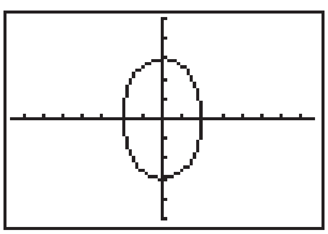
2.



Tmin: [0, 6.28] step: 0.1
$[-7.58, 7.58]$ scl:1 by $[-5, 5]$ scl: 1

2a. $(0, 1)$ **2b.** clockwise

3.



Tmin: [0, 6.28] step: 0.1
$[-7.58, 7.58]$ scl: 1 by $[-5, 5]$ scl: 1
an ellipse

4. The value of a determines the length of the radius of the circle.

5. Each graph is traced out twice.

Page 667 Check for Understanding

1. For the general equation of a conic, A and C have the same sign and $A \neq C$ for an ellipse. A and C have opposite signs for a hyperbola. $A = C$ for a circle. Either $A = 0$ or $C = 0$ for a parabola.

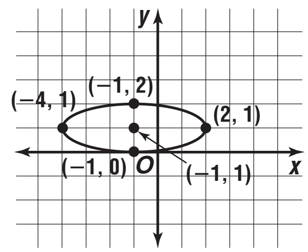
2. $-\infty < t < \infty$

3. Sample answer:
rectangular equation: $y^2 = -x$
parametric equations: $y = t$ $\quad$ $y^2 = -x$
$t^2 = -x$
$-t^2 = x$
$y = t, x = -t^2, -\infty < t < \infty$

4. $A = 1$, $c = 9$; since A and C have the same sign and are not equal, the conic is an ellipse.

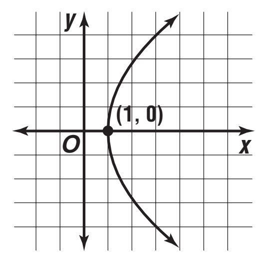
$x^2 + 9y^2 + 2x - 18y + 1 = 0$
$(x^2 + 2x + ?) + 9(y^2 - 2y + ?) = -1 + ? + ?$
$(x^2 + 2x + 1) + 9(y^2 - 2y + 1) = -1 + 1 + 9(1)$
$(x + 1)^2 + 9(y - 1)^2 = 9$
$\frac{(x + 1)^2}{9} + \frac{(y - 1)^2}{1} = 1$

center: $(h, k) = (-1, 1)$
$a^2 = 9$ $\quad$ $b^2 = 1$
$a = 3$ $\quad$ $b = 1$
vertices: $(h \pm a, k) = (-1 \pm 3, 1)$ or $(2, 1), (-4, 1)$
$(h, k \pm b) = (-1, 1 \pm 1)$ or $(-1, 2), (-1, 0)$



5. $A = 1$, $C = 0$; since $C = 0$, the conic is a parabola.

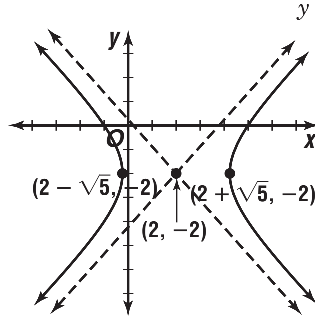
$y^2 - 8x = -8$
$y^2 = 8x - 8$
$y^2 = 8(x - 1)$
vertex: $(h, k) = (1, 0)$
opening: right



6. $A = 1$, $C = -1$; since A and C have different signs, the conic is a hyperbola.

$x^2 - 4x - y^2 - 5 - 4y = 0$
$(x^2 - 4x + ?) - (y^2 + 4y + ?) = 5 + ? + ?$
$(x^2 - 4x + 4) - (y^2 + 4y + 4) = 5 + 4 - 4$
$(x - 2)^2 - (y + 2)^2 = 5$
$\frac{(x - 2)^2}{5} - \frac{(y + 2)^2}{5} = 1$

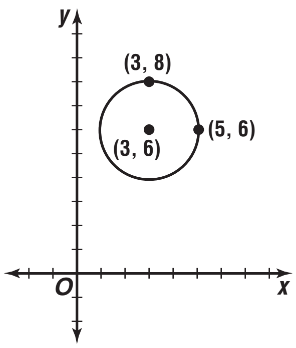
center: $(h, k) = (2, -2)$
$a^2 = 5$
$a = \sqrt{5}$
vertices: $(h \pm a, k) = (2 \pm \sqrt{5}, -2)$
asymptotes: $y - k = \pm \frac{b}{a}(x - h)$
$y - (-2) = \pm \frac{\sqrt{5}}{\sqrt{5}}(x - 2)$
$y + 2 = \pm (x - 2)$



7. $A = 1$, $C = 1$; since $A = C$, the conic is a circle.

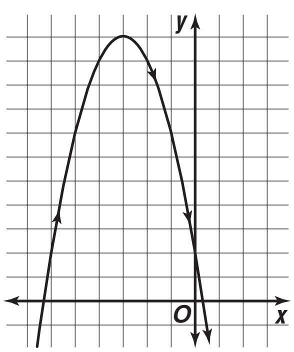
$$x^2 - 6x + y^2 - 12y + 41 = 0$$
$$(x^2 - 6x + ?) + (y^2 - 12y + ?) = -41 + ? + ?$$
$$(x^2 - 6x + 9) + (y^2 - 12y + 36) = -41 + 9 + 36$$
$$(x - 3)^2 + (y - 6)^2 = 4$$

center: $(h, k) = (3, 6)$
radius: $r^2 = 4$
$r = 2$



8. $y = -t^2 - 6t + 2$
$y = -x^2 - 6x + 2$

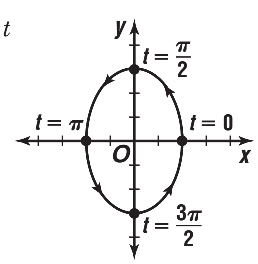
t	x	y	(x, y)
-6	-6	2	$(-6, 2)$
-5	-5	7	$(-5, 7)$
-4	-4	10	$(-4, 10)$
-3	-3	11	$(-3, 11)$
-2	-2	10	$(-2, 10)$
-1	-1	7	$(-1, 7)$
0	0	2	$(0, 2)$



9. $x = 2 \cos t$ $\qquad y = 3 \sin t$

$\frac{x}{2} = \cos t$ $\qquad \frac{y}{3} = \sin t$

$$\cos^2 t + \sin^2 t = 1$$
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$
$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$



t	x	y	(x, y)
0	2	0	$(2, 0)$
$\frac{\pi}{2}$	0	3	$(0, 3)$
π	-2	0	$(-2, 0)$
$\frac{3\pi}{2}$	0	-3	$(0, -3)$

10. Sample answer:
Let $x = t$.
$y = 2x^2 - 5x$
$y = 2t^2 - 5t$
$x = t$, $y = 2t^2 - 5t$, $-\infty \le t \le \infty$

11. Sample answer:

$$x^2 + y^2 = 36$$
$$\frac{x^2}{36} + \frac{y^2}{36} = 1$$
$$\left(\frac{x}{6}\right)^2 + \left(\frac{y}{6}\right)^2 = 1$$

$\cos^2 t + \sin^2 t = 1$

$\left(\frac{x}{6}\right)^2 = \cos^2 t$ $\qquad \left(\frac{y}{6}\right)^2 = \sin^2 t$

$\frac{x}{6} = \cos t$ $\qquad \frac{y}{6} = \sin t$

$x = 6 \cos t$ $\qquad y = 6 \sin t$

$x = 6 \cos t$, $y = 6 \sin t$, $0 \le t \le 2\pi$

12. $x = \frac{t^2}{80}$

$x = \frac{y^2}{80}$

$80x = y^2$

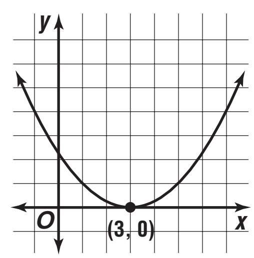
$y^2 = 80x$

Pages 667–669 Exercises

13. $A = 1$, $C = 0$; since $C = 0$, the conic is a parabola.

$$x^2 - 4y - 6x + 9 = 0$$
$$x^2 - 6x + ? = 4y - 9 + ?$$
$$x^2 - 6x + 9 = 4y - 9 + 9$$
$$(x - 3)^2 = 4y$$

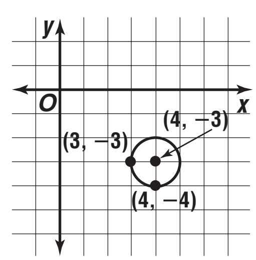
vertex: $(h, k) = (3, 0)$
opening: upward



14. $A = 1$, $C = 1$; since $A = C$, the conic is a circle.

$$x^2 - 8x + y^2 + 6y + 24 = 0$$
$$(x^2 - 8x + ?) + (y^2 + 6y + ?) = -24 + ? + ?$$
$$(x^2 - 8x + 16) + (y^2 + 6y + 9) = -24 + 16 + 9$$
$$(x - 4)^2 + (y + 3)^2 = 1$$

center: $(h, k) = (4, -3)$
radians: $r^2 = 1$
$r = 1$



15. $A = -1$, $C = 3$; since A and C have different signs, the conic is a hyperbola.

$$x^2 - 3y^2 + 2x - 24y - 41 = 0$$
$$-x^2 + 3y^2 - 2x + 24y + 41 = 0$$
$$3(y^2 + 8y + ?) - (x^2 + 2x + ?) = -41 + ? + ?$$
$$3(y^2 + 8y + 16) - (x^2 + 2x + 1) = -41 + 3(16) - 1$$
$$3(y + 4)^2 - (x + 1)^2 = 6$$
$$\frac{(y + 4)^2}{2} - \frac{(x + 1)^2}{6} = 1$$

center: $(h, k) = (-1, -4)$

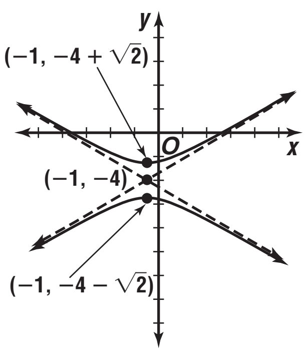
$a^2 = 2$

$a = \sqrt{2}$

vertices: $(h, k + a) = (-1, -4 \pm \sqrt{2})$

asymptotes: $y - k = \pm \frac{a}{b}(x - h)$

$$y - (-4) = \pm \frac{\sqrt{2}}{\sqrt{6}}[x - (-1)]$$
$$y + 4 = \pm \frac{\sqrt{3}}{3}(x + 1)$$



16. $A = 9$, $C = 25$; since A and C have the same sign and are not equal, the conic is an ellipse.

$$9x^2 + 25y^2 - 54x - 50y - 119 = 0$$
$$9(x^2 - 6x + ?) + 25(y^2 - 2y + ?) = 119 + ? + ?$$
$$9(x^2 - 6x + 9) + 25(y^2 - 2y + 1) = 119 + 9(9) + 25(1)$$
$$9(x - 3)^2 + 25(y - 1)^2 = 225$$
$$\frac{(x - 3)^2}{25} + \frac{(y - 1)^2}{9} = 1$$

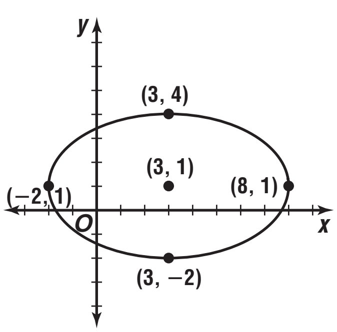
center: $(h, k) = (3, 1)$

$a^2 = 25$ $\quad$ $b^2 = 9$

$a = 5$ $\quad$ $b = 3$

vertices: $(h \pm a, k) = (3 \pm 5, 1)$ or $(8,1), (-2, 1)$

$(h, k \pm b) = (3, 1 \pm 3)$ or $(3, 4), (3, -2)$

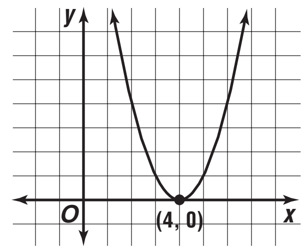


17. $A = 1$, $C = 0$; since $C = 0$, the conic is a parabola.

$$x^2 = y + 8x - 16$$
$$x^2 - 8x + ? = y - 16 + ?$$
$$x^2 - 8x + 16 = y - 16 + 16$$
$$(x - 4)^2 = y$$

vertex: $(h, k) = (4, 0)$

opening: upward



18. $A = C = D = E = 0$; the conic is a hyperbola.

$2xy = 3$

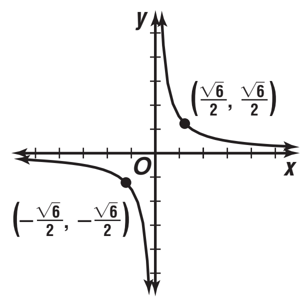
$xy = \frac{3}{2}$

quadrants: I and III

transverse axis: $y = x$

vertices: $\left(\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}\right)$ or $\left(\sqrt{\frac{6}{2}}, \sqrt{\frac{6}{2}}\right)$,

$\left(-\sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}\right)$ or $\left(-\frac{\sqrt{6}}{2}, -\frac{\sqrt{6}}{2}\right)$



19. $A = 5$, $C = 2$; since A and C have the same sign and are not equal, the conic is an ellipse.

$$5x^2 + 2y^2 - 40x - 20y + 110 = 0$$
$$5(x^2 - 8x + ?) + 2(y^2 - 10y + ?) = -110 + ? + ?$$
$$5(x^2 - 8x + 16) + 2(y^2 - 10y + 25) = -110 + 5(16) + 2(25)$$
$$5(x - 4)^2 + 2(y - 5)^2 = 20$$
$$\frac{(x - 4)^2}{4} + \frac{(y - 5)^2}{10} = 1$$
$$\frac{(y - 5)^2}{10} + \frac{(x - 4)^2}{4} = 1$$

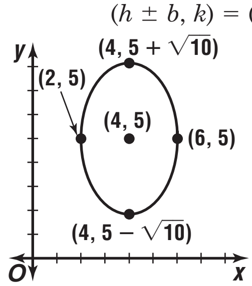
center: $(h, k) = (4, 5)$

$a^2 = 10$ $\quad$ $b^2 = 4$

$a = \sqrt{10}$ $\quad$ $b = 2$

vertices: $(h, k \pm a) = (4, 5 \pm \sqrt{10})$

$(h \pm b, k) = (4 \pm 2, 5)$ or $(6, 5), (2, 5)$



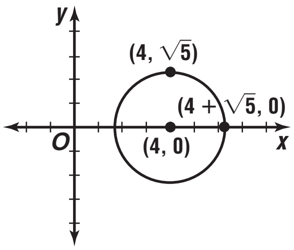
20. $A = 1$, $C = 1$; since $A = C$, the conic is a circle.

$$x^2 - 8x + 11 = -y^2$$
$$(x^2 - 8x + ?) + y^2 = -11 + ?$$
$$(x^2 - 8x + 16) + y^2 = -11 + 16$$
$$(x - 4)^2 + y^2 = 5$$

center: $(h, k) = (4, 0)$

radius: $r^2 = 5$

$r = \sqrt{5}$



21. $A = -9$, $C = 8$; since A and C have different signs, the conic is a hyperbola.

$$8y^2 - 9x^2 - 16y + 36x - 100 = 0$$
$$8(y^2 - 2y + ?) - 9(x^2 - 4x + ?) = 100 + ? + ?$$
$$8(y^2 - 2y + 1) - 9(x^2 - 4x + 4) = 100 + 8(1) - 9(4)$$
$$8(y - 1)^2 - 9(x - 2)^2 = 72$$
$$\frac{(y-1)^2}{9} - \frac{(x-2)^2}{8} = 1$$

center: $(h, k) = (2, 1)$

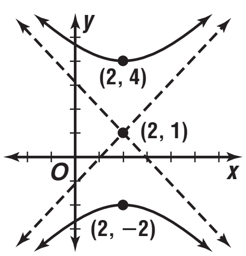
$a^2 = 9$

$a = 3$

vertices: $(h, k \pm a) = (2, 1 \pm 3)$ or $(2, 4)$, $(2, -2)$

asymptotes: $y - k = \pm \frac{a}{b}(x - h)$

$$y - 1 = \pm \frac{3}{2\sqrt{2}}(x - 2)$$
$$y - 1 = \pm \frac{3\sqrt{2}}{4}(x - 2)$$

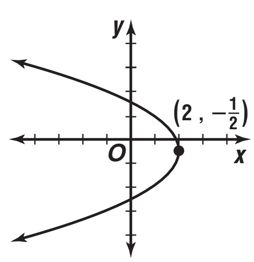


22. $A = 0$, $C = 4$; since $A = 0$, the conic is a parabola.

$$4y^2 + 4y + 8x = 15$$
$$4(y^2 + y + ?) = -8x + 15 + ?$$
$$4\left(y^2 + y + \frac{1}{4}\right) = -8x + 15 + 4\left(\frac{1}{4}\right)$$
$$4\left(y + \frac{1}{2}\right)^2 = -8x + 16$$
$$\left(y + \frac{1}{2}\right)^2 = -2x + 4$$
$$\left(y + \frac{1}{2}\right)^2 = -2(x - 2)$$

vertex: $(h, k) = \left(2, -\frac{1}{2}\right)$

opening: left



23.

$$-4y^2 + 10x = 16y - x^2 - 5$$
$$x^2 - 4y^2 + 10x - 16y + 5 = 0$$

$A = 1$, $C = -4$; since A and C have different signs, the conic is a hyperbola.

$$x^2 - 4y^2 + 10x - 16y + 5 = 0$$
$$(x^2 + 10x + ?) - 4(y^2 + 4y + ?) = -5 + ? + ?$$
$$(x^2 + 10x + 25) - 4(y^2 + 4y + 4) = -5 + 25 - 4(4)$$
$$(x + 5)^2 - 4(y + 2)^2 = 4$$
$$\frac{(x+5)^2}{4} - \frac{(y+2)^2}{1} = 1$$

center: $(h, k) = (-5, -2)$

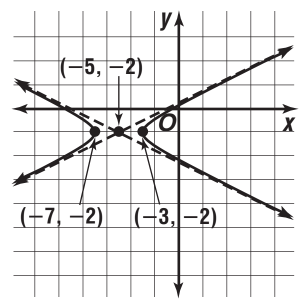
$a^2 = 4$

$a = 2$

vertices: $(h \pm a, k) = (-5 \pm 2, -2)$ or $(-3, -2)$, $(-7, -2)$

asymptotes: $y - k = \pm \frac{b}{a}(x - h)$

$$y - (-2) = \pm \frac{1}{2}[x - (-5)]$$
$$y + 2 = \pm \frac{1}{2}(x + 5)$$



24.

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$
$$2x^2 + 0 + 2y^2 + (-8)x + 12y + 6 = 0$$
$$2x^2 + 2y^2 - 8x + 12y = -6$$

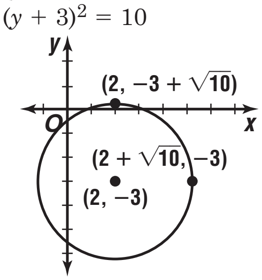
A = C; circle

$$2(x^2 - 4x + ?) + 2(y^2 + 6y + ?) = 6 + ? + ?$$
$$2(x^2 - 4x + 4) + 2(y^2 + 6y + 6y + 9) =$$
$$-6 + 2(4) + 2(9)$$
$$2(x - 2)^2 + 2(y + 3)^2 = 20$$
$$(x - 2)^2 + (y + 3)^2 = 10$$

center: $(h, k) = (2, -3)$

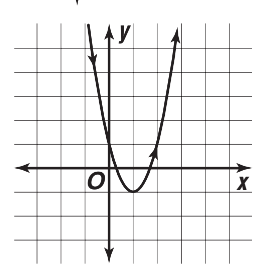
radius: $r^2 = 10$

$r = \sqrt{10}$



25. $y = 2t^2 - 4t + 1$

$y = 2x^2 - 4x + 1$



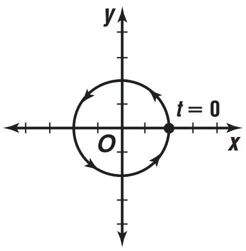
t	x	y	(x, y)
-1	-1	7	$(-1, 7)$
0	0	1	$(0, 1)$
1	1	-1	$(1, -1)$
2	2	7	$(2, 7)$

26. $x = \cos 2t \qquad y = \sin 2t$

$\cos^2 2t + \sin^2 2t = 1$

$x^2 + y^2 = 1$

t	x	y	(x, y)
0	1	0	(1, 0)
$\frac{\pi}{2}$	0	1	(0, 1)
π	-1	0	$(-1, 0)$
$\frac{3\pi}{2}$	0	-1	$(0, -1)$



27. $x = -\cos t \qquad y = \sin t$

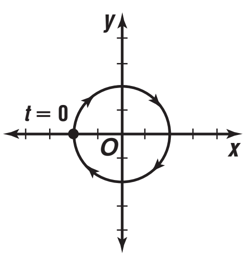
$-x = \cos t$

$\cos^2 t + \sin^2 t = 1$

$(-x)^2 + y^2 = 1$

$x^2 + y^2 = 1$

t	x	y	(x, y)
0	-1	0	$(-1, 0)$
$\frac{\pi}{2}$	0	1	(0, 1)
π	1	0	(1, 0)
$\frac{3\pi}{2}$	0	-1	$(0, -1)$



28. $x = 3 \sin t \qquad y = 2 \cos t$

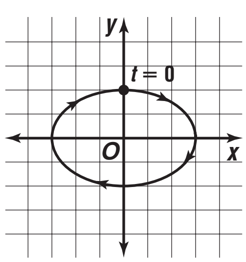
$\frac{x}{3} = \sin t \qquad \frac{y}{2} = \cos t$

$\cos^2 t + \sin^2 t = 1$

$\left(\frac{y}{2}\right)^2 + \left(\frac{x}{3}\right)^2 = 1$

$\frac{y^2}{4} + \frac{x^2}{9} = 1$

$\frac{x^2}{9} + \frac{y^2}{4} = 1$



t	x	y	(x, y)
0	2	0	(0, 2)
$\frac{\pi}{2}$	3	0	(3, 0)
π	0	-2	$(0, -2)$
$\frac{3\pi}{2}$	-3	0	$(-3, 0)$

29. $x = -\sin 2t \qquad y = 2 \cos 2t$

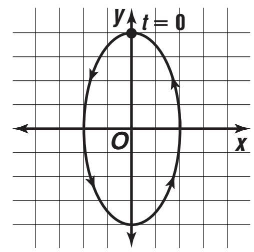
$-x = \sin 2t \qquad \frac{y}{2} = \cos 2t$

$\cos^2 2t + \sin^2 2t = 1$

$\left(\frac{y}{2}\right)^2 + (-x)^2 = 1$

$\frac{y^2}{4} + x^2 = 1$

$x^2 + \frac{y^2}{4} = 1$



t	x	y	(x, y)
0	0	2	(0, 2)
$\frac{\pi}{4}$	-1	0	$(-1, 0)$
$\frac{\pi}{2}$	0	-2	$(0, -2)$
$\frac{3\pi}{4}$	1	0	(1, 0)

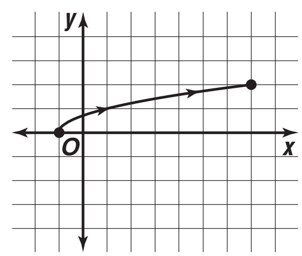
30. $x = 2t - 1$

$x + 1 = 2t$

$\frac{x + 1}{2} = t$

$y = \sqrt{t}$

$y = \sqrt{\frac{x + 1}{2}}$



t	x	y	(x, y)
0	-1	0	$(-1, 0)$
1	1	1	(1, 1)
2	3	$\sqrt{2}$	$(3, \sqrt{2})$
3	5	$\sqrt{3}$	$(5, \sqrt{3})$
4	7	2	(7, 2)

31. $x = -3 \cos 2t \qquad y = 3 \sin 2t$

$-\frac{x}{3} = \cos 2t \qquad \frac{y}{3} = \sin 2t$

$\cos^2 2t + \sin^2 2t = 1$

$\left(-\frac{x}{3}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

$\frac{x^2}{9} + \frac{y^2}{9} = 1$

$x^2 + y^2 = 9$

32. Sample answer:

$x^2 + y^2 = 25$

$\frac{x^2}{25} + \frac{y^2}{25} = 1$

$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$

$\cos^2 t + \sin^2 t = 1$

$\left(\frac{x}{5}\right)^2 = \cos^2 t \qquad \left(\frac{y}{5}\right)^2 = \sin^2 t$

$\frac{x}{5} = \cos t \qquad \frac{y}{5} = \sin t$

$x = 5 \cos t \qquad y = 5 \sin t$

$x = 5 \cos t,\ y = 5 \sin t,\ 0 \le t \le 2\pi$

33. Sample answer:

$$x^2 + y^2 - 16 = 0$$
$$x^2 + y^2 = 16$$
$$\frac{x^2}{16} + \frac{y^2}{16} = 1$$
$$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$
$$\cos^2 t + \sin^2 t = 1$$

$\left(\frac{x}{4}\right)^2 = \cos^2 t$ $\qquad$ $\left(\frac{y}{4}\right)^2 = \sin^2 t$

$\frac{x}{4} = \cos t$ $\qquad$ $\frac{y}{4} = \sin t$

$x = 4 \cos t$ $\qquad$ $y = 4 \sin t$

$x = 4 \cos t, y = 4 \sin t, 0 \leq t \leq 2\pi$

34. Sample answer:

$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$$
$$\cos^2 t + \sin^2 t = 1$$

$\left(\frac{x}{2}\right)^2 = \cos^2 t$ $\qquad$ $\left(\frac{y}{5}\right)^2 = \sin^2 t$

$\frac{x}{2} = \cos t$ $\qquad$ $\frac{y}{5} = \sin t$

$x = 2 \cos t$ $\qquad$ $y = 5 \sin t$

$x = 2 \cos t, y = 5 \sin t, 0 \leq t \leq 2\pi$

35. Sample answer:

$$\frac{y^2}{16} + x^2 = 1$$
$$x^2 + \left(\frac{y}{4}\right)^2 = 1$$
$$\cos^2 t + \sin^2 t = 1$$

$x^2 = \cos^2 t$ $\qquad$ $\left(\frac{y}{4}\right)^2 = \sin^2 t$

$x = \cos t$ $\qquad$ $\frac{y}{4} = \sin t$

$y = 4 \sin t$

$x = \cos t, y = 4 \sin t, 0 \leq t \leq 2\pi$

36. Sample answer:

Let $x = t$.

$y = x^2 - 4x + 7$

$y = t^2 - 4t + 7$

$x = t, y = t^2 - 4t + 7, -\infty < t < \infty$

37. Sample answer:

Let $y = t$.

$x = y^2 + 2y - 1$

$x = t^2 + 2t - 1$

$x = t^2 + 2t - 1, y = t, -\infty < t < \infty$

38. Sample answer:

Let $y = t$.

$$(y + 3)^2 = 4(x - 2)$$
$$(t + 3)^2 = 4(x - 2)$$
$$0.25(t + 3)^2 = x - 2$$
$$0.25(t + 3)^2 + 2 = x$$

$x = 0.25(t + 3)^2 + 2, y = t, -\infty < t < \infty$

39a. Answers will vary. Sample answers:

Let $x = t$.

$x = \sqrt{y}$

$t = \sqrt{y}$

$t^2 = y$

$x = t, y = t^2, t \geq 0$

Let $y = t$.

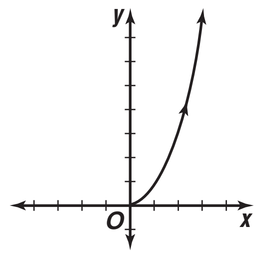
$x = \sqrt{y}$

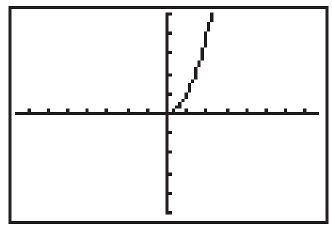
$x = \sqrt{t}$

$x = \sqrt{t}, y = t, t \geq 0$

39b.

t	x	y	(x, y)
0	0	0	(0, 0)
1	1	1	(1, 1)
2	2	4	(2, 4)
3	3	9	(3, 9)





Tmin: [0, 5] step: 0.1

[−7.58, 7.58] scl: 1 by [−5, 5] scl: 1

39c. yes

39d. There is usually more than one parametric representation for the graph of a rectangular equation.

40a. a circle with center (0, 0) and radius 6 feet

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 0)^2 + (y - 0)^2 = 6^2$

$x^2 + y^2 = 36$

40b. Sample answer:

$$x^2 + y^2 = 36$$
$$\frac{x^2}{36} + \frac{y^2}{36} = 1$$
$$\left(\frac{x}{6}\right)^2 + \left(\frac{y}{6}\right)^2 = 1$$

$\sin^2 (\omega t) + \cos^2 (\omega t) = 1$

Since the paddlewheel completes a revolution in 2 seconds, the period is $\frac{2\pi}{\omega} = 2$, so $\omega = \pi$.

$\sin^2 (\pi t) + \cos^2(\pi t) = 1$

$\left(\frac{x}{6}\right)^2 = \sin^2(\pi t)$ $\qquad$ $\left(\frac{y}{6}\right)^2 = \cos^2 (\pi t)$

$\frac{x}{6} = \sin(\pi t)$ $\qquad$ $\frac{y}{6} = \cos(\pi t)$

$x = 6 \sin(\pi t)$ $\qquad$ $y = 6 \cos (\pi t)$

$x = 6 \sin (\pi t), y = 6 \cos (\pi t), 0 \leq t \leq 2$

40c. $C = 2\pi r$

$C = 2\pi 6$

$C \approx 37.7$ ft

The paddlewheel makes 1 revolution, or moves 37.7 ft in 2 seconds.

$\frac{37.7 \text{ ft}}{2 \text{ s}} \cdot 60 \text{ s} = 1131 \text{ ft}$

The paddlewheel moves about 1131 ft in 1 minute.

41a. $A = 2$, $C = 5$; since A and C have the same sign and $A \neq C$, the graph is an ellipse.

$$2x^2 + 5y^2 = 0$$
$$5y^2 = -2x$$
$$y^2 = -\frac{2}{5}x$$
$$y = \sqrt{-\frac{2}{5}x}$$

This equation is true for $(x, y) = (0, 0)$.
The graph is a point at (0, 0); the equation is that of a degenerate ellipse.

41b. $A = 1$, $C = 1$; since $A = C$, the graph is a circle.

$$x^2 + y^2 - 4x - 6y + 13 = 0$$
$$(x^2 - 4x + ?) + (y^2 - 6y + ?) = -13$$
$$(x^2 - 4x + 4) + (y^2 - 6y + 9) = -13 + 4 + 9$$
$$(x - 2)^2 + (y - 3)^2 = 0$$

center: $(h, k) = (2, 3)$
radius: 0
The graph is a point at (2, 3); the equation is that of a degenerate circle.

41c. $A = -9$, $C = 1$; since A and C have different signs, the graph is a hyperbola.

$$y^2 - 9x^2 = 0$$
$$y^2 = 9x^2$$
$$y = \pm 3x$$

The graph is two intersecting lines $y = \pm 3x$; the equation is that of a degenerate hyperbola.

42. The substitution for x must be a function that allows x to take on all of the values stipulated by the domain of the rectangular equation. The domain of $y = x^2 - 5$ is all real numbers, but using a substitution of $x = t^2$ would only allow for values of x such that $x \geq 0$.

43a. center: $(h, k) = (0, 0)$

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 0)^2 + (y - 0)^2 = 6$$
$$x^2 + y^2 = 36$$

43b.

$$x^2 + y^2 = 36$$
$$\frac{x^2}{36} + \frac{y^2}{36} = 1$$
$$\left(\frac{x}{6}\right)^2 + \left(\frac{y}{6}\right)^2 = 1$$
$$\sin^2 t + \cos^2 t = 1$$

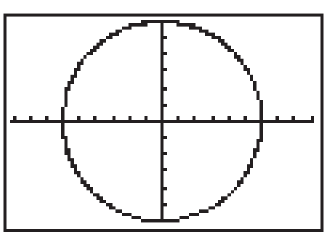
$\left(\frac{x}{6}\right)^2 = \sin^2 t$ $\qquad$ $\left(\frac{y}{6}\right)^2 = \cos^2 t$

$\frac{x}{6} = \sin t$ $\qquad$ $\frac{y}{6} = \cos t$

$x = 6 \sin t$ $\qquad$ $y = 6 \cos t$

$x = 6 \sin t$, $y = 6 \cos t$
Since the second hand makes 2 revolutions, $0 \leq t \leq 4\pi$.

43c.



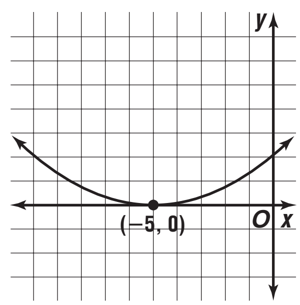
Tmin: $[0, 4\pi]$ step: 0.1
$[-9.10, 9.10]$ scl:1 by $[-6, 6]$ scl:1

44. After drawing a vertical line through (x, y) and a horizontal line through the endpoint opposite (x, y), two right triangles are formed. Both triangles contain an angle t, since corresponding angles are congruent when two parallel lines are cut by a transversal. Using the larger triangle, $\cos t = \frac{x}{a}$ or $x = a \cos t$. Using the smaller triangle, $\sin t = \frac{y}{b}$ or $y = b \sin t$.

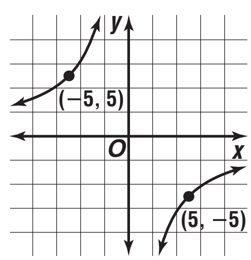
45.

$$x^2 - 12y + 10x = -25$$
$$x^2 + 10x + ? = 12y - 25 + ?$$
$$x^2 + 10x + 25 = 12y - 25 + 25$$
$$(x + 5)^2 = 12y$$

vertex: $(h, k) = (-5, 0)$
$4p = 12$
$p = 3$
focus: $(h, k + p) = (-5, 0 + 3)$ or $(-5, 3)$
axis of symmetry: $x = h$
$x = -5$
directrix: $y = k - p$
$y = 0 - 3$
$y = -3$



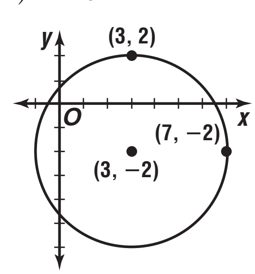
46. $c = -25$
quadrants: II and IV
transverse axis: $y = -x$
vertices: $xy = -25$ $\qquad$ $xy = -25$
$5(-5) = -25$ $\qquad$ $-5(5) = -25$
$(5, -5)$ $\qquad$ $(-5, 5)$

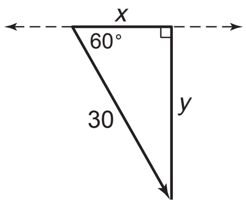


47.

$$3x^2 + 3y^2 - 18x + 12y = 9$$
$$3(x^2 - 6x + ?) + 3(y^2 + 4y + ?) = 9 + ? + ?$$
$$3(x^2 - 6x + 9) + 3(y^2 + 4y + 4) = 9 + 3(9) - 3(4)$$
$$3(x - 3)^2 + 3(y + 2)^2 = 48$$
$$(x - 3)^2 + (y + 2)^2 = 16$$

center: $(h, k) = (3, -2)$
radians: $r^2 = 16$
$r = 4$



48. 

$\cos 60° = \frac{x}{30}$ $\qquad$ $\sin 60° = \frac{y}{30}$

$x = 30 \cos 60°$ $\qquad$ $y = 30 \sin 60°$

$x = 15$ lb $\qquad$ $y = 15\sqrt{3}$ lb

49. $y = -0.13x + 37.8$

$0.13x + y - 37.8 = 0$

$A = 0.13, B = 1, C = -37.8$

Car 1: $(x_1, y_1) = (135, 19)$

$d_1 = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$

$d_1 = \frac{0.13(135) + 1(19) + (-37.8)}{\pm\sqrt{(0.13)^2 + 1^2}}$

$d_1 \approx -1.24$

The point (135, 19) is about 1 unit from the line $y = -0.13x + 37.8$.

Car 2: $(x_2, y_2) = (245, 16)$

$d_2 = \frac{Ax_2 + By_2 + C}{\pm\sqrt{A^2 + B^2}}$

$d_2 = \frac{0.13(245) + 1(16) + (-37.8)}{\pm\sqrt{(0.13)^2 + 12}}$

$d_2 \approx 9.97$

The point (245, 16) is about 10 units from the line $y = -0.13x + 37.8$.

Car 1: the point (135, 19) is about 9 units closer to the line $y = -0.13x + 37.8$ than the point (245, 16).

50. Let $\theta = \text{Sin}^{-1}\frac{1}{2}$.

$\text{Sin}\,\theta = \frac{1}{2}$

$\theta = 30$

$\sin\left(2\,\text{Sin}^{-1}\frac{1}{2}\right) = \sin(2\theta)$

$= \sin(2 \cdot 30)$

$= \sin 60$

$= \frac{\sqrt{3}}{2}$

51. $s = \frac{1}{2}(a + b + c)$

$s = \frac{1}{2}(48 + 32 + 44)$

$s = 62$

$K = \sqrt{s(s - a)(s - b)(s - c)}$

$K = \sqrt{62(62 - 48)(62 - 32)(62 - 44)}$

$K = \sqrt{468720}$

$K \approx 685$ units^2

52. $\sqrt{2y - 3} - \sqrt{2y + 3} = -1$

$\sqrt{2y - 3} = \sqrt{2y + 3} - 1$

$2y - 3 = 2y + 3 - 2\sqrt{2y + 3} + 1$

$-7 = -2\sqrt{2y + 3}$

$\frac{7}{2} = \sqrt{2y + 3}$

$\frac{49}{4} = 2y + 3$

$\frac{37}{4} = 2y$

$\frac{37}{8} = y$

53. $y = kxz$ $\qquad$ $y = kxz$

$16 = k(5)(2)$ $\qquad$ $y = 1.6(8)(3)$

$1.6 = k$ $\qquad$ $y = 38.4$

54. $\begin{vmatrix} 5 & 9 \\ 7 & -3 \end{vmatrix} = 5(-3) - 7(9)$

$= -78$

Yes, an inverse exists since the determinant of the matrix $\neq 0$.

55. $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7 - 4}{3 - (-6)}$

$= \frac{1}{3}$

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{1}{3}(x + 6)$ or $y - 7 = \frac{1}{3}(x - 3)$

$y = mx + b$ $\qquad$ $y = mx + b$

$4 = \frac{1}{3}(-6) + b$ $\qquad$ $y = \frac{1}{3}x + 6$

$6 = b$

56. (1 # 4) @ (2 # 3) = 1 @ 2

= 2

The correct choice is *B*.

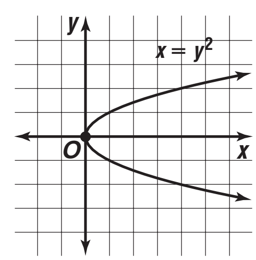
10-7 Transformation of Conics

Pages 674–675 Check for Understanding

1. Sample answers:

$(h, k) = (0, 0)$

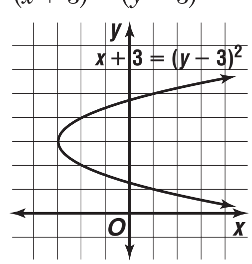
$x = y^2$



$(h, k) = (-3, 3)$

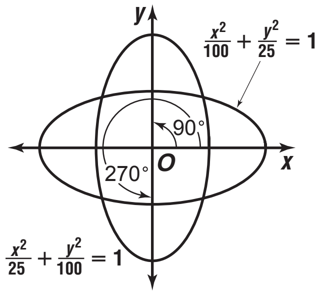
$(x - h) = (y - k)^2$

$(x + 3) = (y - 3)^2$



2. Replace x with $x' \cos 30° + y' \sin 30°$ or $\frac{\sqrt{3}}{2}x' + \frac{1}{2}y'$.

Replace y with $-x' \sin 30° + y' \cos 30°$ or $-\frac{1}{2}x', + \frac{\sqrt{2}}{2}y'$.

3. 

90° or 270°

4. Ebony; $B^2 - 4AC = \left(6\sqrt{3}\right)^2 - 4(7)(13) < 0$ and $A \neq C$

5. $B^2 - 4AC = 0 - 4(1)(1)$
$= -4$
$A = C = 1$; circle

$$(x - h)^2 + (y - k)^2 = 7$$
$$(x - 3)^2 + (y - 2)^2 = 7 \qquad (h, k) = (3, 2)$$
$$x^2 - 6x + 9 + y^2 - 4y + 4 = 7$$
$$x^2 + y^2 - 6x - 4y + 6 = 0$$

6. $B^2 - 4AC = 0 - 4(2)(0)$
$= 0$
parabola

$$y = 2x^2 - 7x + 5$$
$$y - 5 = 2x^2 - 7x$$
$$y - 5 = 2\left(x^2 - \tfrac{7}{2}x\right)$$
$$y - 5 + 2\left(\tfrac{7}{4}\right)^2 = 2\left[x^2 - \tfrac{7}{2}x + \left(\tfrac{7}{4}\right)^2\right]$$
$$y - 5 + \tfrac{49}{8} = 2\left(x - \tfrac{7}{4}\right)^2$$
$$y + \tfrac{9}{8} = 2\left(x - \tfrac{7}{4}\right)^2$$
$$y + \tfrac{9}{8} - k = 2\left(x - \tfrac{7}{4} - h\right)^2$$
$$y + \tfrac{9}{8} - 5 = 2\left(x - \tfrac{7}{4} + 4\right)^2 \qquad (h, k) = (-4, 5)$$
$$y - \tfrac{31}{8} = 2\left(x + \tfrac{9}{4}\right)^2$$
$$y = \tfrac{31}{8} = 2\left(x^2 + \tfrac{18}{4}x + \tfrac{81}{16}\right)$$
$$y - \tfrac{31}{8} = 2x^2 + 9x + \tfrac{81}{8}$$
$$0 = 2x^2 + 9x - y + 14$$
$$2x^2 + 9x - y + 14 = 0$$

7. $B^2 - 4AC = 0 - 4(1)(-1)$
$= 4$
hyperbola

$$x^2 - y^2 = 9$$
$$(x' \cos 60° + y' \text{ sub } 60°)^2 -$$
$$(-x' \sin 60° + y' \cos 60°)^2 = 9$$
$$\left(\tfrac{1}{2}x' + \tfrac{\sqrt{3}}{2}y'\right)^2 - \left(-\tfrac{\sqrt{3}}{2}x' + \tfrac{1}{2}y'\right)^2 = 9$$
$$\tfrac{1}{4}(x')^2 + \tfrac{\sqrt{3}}{2}x'y' + \tfrac{3}{4}(y')^2 -$$
$$\left[\tfrac{3}{4}(x')^2 - \tfrac{\sqrt{3}}{2}x'y' + \tfrac{1}{4}(y')^2\right] = 9$$
$$-\tfrac{1}{2}(x')^2 + \sqrt{3}x'y' + \tfrac{1}{2}(y')^2 = 9$$
$$(x')^2 - 2\sqrt{3}x'y' - (y')^2 = -18$$
$$(x')^2 - 2\sqrt{3}x'y' - (y')^2 + 18 = 0$$

8. $B^2 - 4AC = 0 - 4(1)(1)$
$= -4$
$A = C = 1$; circle

$$x^2 - 5x + y^2 = 3$$
$$\left(x' \cos \tfrac{\pi}{4} + y' \sin \tfrac{\pi}{4}\right)^2 - 5\left(x' \cos \tfrac{\pi}{4} + y' \sin \tfrac{\pi}{4}\right)$$
$$+ \left(-x' \sin \tfrac{\pi}{4} + y' \cos \tfrac{\pi}{4}\right)^2 = 3$$
$$\left(\tfrac{\sqrt{2}}{2}x' + \tfrac{\sqrt{2}}{2}y'\right)^2 - 5\left(\tfrac{\sqrt{2}}{2}x' + \tfrac{\sqrt{2}}{2}y'\right)$$
$$+ \left(-\tfrac{\sqrt{2}}{2}x' + \tfrac{\sqrt{2}}{2}y'\right)^2 = 3$$
$$\tfrac{1}{2}(x')^2 + x'y' + \tfrac{1}{2}(y')^2 - \tfrac{5\sqrt{2}}{2}x' - \tfrac{5\sqrt{2}}{2}y'$$
$$+ \tfrac{1}{2}(x')^2 - x'y' + \tfrac{1}{2}(y')^2 = 3$$
$$(x')^2 + (y')^2 - \tfrac{5\sqrt{2}}{2}x' - \tfrac{5\sqrt{2}}{2}y' = 3$$
$$2(x')^2 + 2(y')^2 - 5\sqrt{2}x' - 5\sqrt{2}y' = 6$$
$$2(x')^2 + 2(y')^2 - 5\sqrt{2}x' - 5\sqrt{2}y' - 6 = 0$$

9. $B^2 - 4AC = 4^2 - 4(9)(4)$
$= -128$
A ≠ C; ellipse

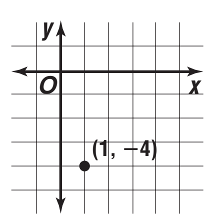
$$\tan 2\theta = \tfrac{B}{A - C}$$
$$\tan 2\theta = \tfrac{4}{9 - 4}$$
$$\tan 2\theta = 0.8$$
$$2\theta \approx 38.65980825°$$
$$\theta \approx 19°$$

10. $B^2 - 4AC = 5^2 - 4(8)(-4)$
$= 153$
hyperbola

$$\tan 2\theta = \tfrac{B}{A - C}$$
$$\tan 2\theta = \tfrac{5}{8 - (-4)}$$
$$\tan 2\theta \approx 0.416666667$$
$$2\theta \approx 22.61986495°$$
$$\theta \approx 11°$$

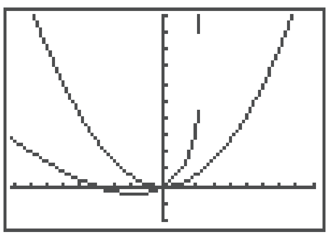
11.
$$3(x - 1)^2 + 4(y + 4)^2 = 0$$
$$3(x^2 - 2x + 1) + 4(y^2 + 8y + 16) = 0$$
$$3x^2 - 6x + 3 + 4y^2 + 32y + 64 = 0$$
$$\underset{\substack{\uparrow \\ a}}{4y^2} + \underset{\substack{\uparrow \\ b}}{32y} + \underbrace{(3x^2 - 6x + 67)}_{\substack{\uparrow \\ c}} = 0$$
$$y = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$y = \frac{-32 \pm \sqrt{32^2 - 4(4)(3x^2 - 6x + 67)}}{2(4)}$$
$$y = \frac{-32 \pm \sqrt{-48x^2 + 96x - 48}}{8}$$
$$y = \frac{-32 \pm \sqrt{-48(x - 1)^2}}{8}$$

$x = 1$, $y = -4$; point



12a. $$y = \frac{1}{6}x^2$$
$$-x' \sin 30° + y' \cos 30° = \frac{1}{6}(x' \cos 30° + y' \sin 30°)^2$$
$$-\frac{1}{2}x' + \frac{\sqrt{3}}{2}y' = \frac{1}{6}\left(\frac{\sqrt{3}}{2}x' + \frac{1}{2}y'\right)^2$$
$$-\frac{1}{2}x' + \frac{\sqrt{3}}{2}y' = \frac{1}{6}\left[\frac{3}{4}(x')^2 + \frac{2\sqrt{3}}{4}x'y' + \frac{1}{4}(y')^2\right]$$
$$-\frac{1}{2}x' + \frac{\sqrt{3}}{2}y' = \frac{1}{8}(x')^2 + \frac{\sqrt{3}}{12}x'y' + \frac{1}{24}(y')^2$$
$$-12x' + 12\sqrt{3}y' = 3(x')^2 + 2\sqrt{3}x'y' + (y')^2$$
$$0 = 3(x')^2 + 2\sqrt{3}x'y' + (y')^2 + 12x' - 12\sqrt{3}y'$$
$$3(x') + 2\sqrt{3}x'y' + (y')^2 + 12x' - 12\sqrt{3}y' = 0$$

12b. $$3x^2 + 2\sqrt{3}xy + y^2 + 12x - 12\sqrt{3}y = 0$$
$$\underbrace{1}_{a}y^2 + \underbrace{(2\sqrt{3}x - 12\sqrt{3})}_{b}y + \underbrace{(3x^2 + 12x)}_{c} = 0$$
$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$y = \frac{-(2\sqrt{3}x - 12\sqrt{3}) \pm \sqrt{(2\sqrt{3}x - 12\sqrt{3})^2 - 4(1)(3x^2 + 12x)}}{2(1)}$$
$$y = -\sqrt{3}x + 6\sqrt{3} \pm \frac{\sqrt{12x^2 - 144x + 432 - 12x^2 - 48x}}{2}$$
$$y = -\sqrt{3}x + 6\sqrt{3} \pm \frac{\sqrt{-192x + 432}}{2} \text{ and } y = \frac{1}{6}x^2$$



Pages 675–677 Exercises

13. $$B^2 - 4AC = 0 - 4(3)(0) = 0$$
parabola
$$y = 3x^2 - 2x + 5$$
$$y - 5 = 3x^2 - 2x$$
$$y - 5 = 3\left(x^2 - \frac{2}{3}x\right)$$
$$y - 5 + 3\left(\frac{1}{3}\right)^2 = 3\left[x^2 - \frac{2}{3}x + \left(\frac{1}{3}\right)^2\right]$$
$$y - \frac{14}{3} = 3\left(x - \frac{1}{3}\right)^2$$
$$y - \frac{14}{3} - k = 3\left(x - \frac{1}{3} - h\right)^2$$
$$y - \frac{14}{3} + 3 = 3\left(x - \frac{1}{3} - 2\right)^2 \qquad (h, k) = (2, -3)$$
$$y - \frac{5}{3} = 3\left(x - \frac{7}{3}\right)^2$$
$$y - \frac{5}{3} = 3\left(x^2 - \frac{14}{3}x + \frac{49}{9}\right)$$
$$y - \frac{5}{3} = 3x^2 - 14x + \frac{49}{3}$$
$$0 = 3x^2 - 14x - y + 18$$
$$3x^2 - 14x - y + 18 = 0$$

14. $$B^2 - 4AC = 0 - 4(4)(5) = -80$$
$A \neq C$; ellipse
$$4x^2 + 5y^2 = 20$$
$$4(x - h)^2 + 5(y - k)^2 = 20$$
$$4(x - 5)^2 + 5(y + 6)^2 = 20 \qquad (h, k) = (5, -6)$$
$$4(x^2 - 10x + 25) + 5(y^2 + 12y + 36) = 20$$
$$4x^2 - 40x + 100 + 5y^2 + 60y + 180 = 20$$
$$4x^2 + 5y^2 - 40x + 60y + 260 = 0$$

15. $$B^2 - 4AC = 0 - 4(3)(1) = -12$$
$A \neq C$; ellipse
$$3x^2 + y^2 = 9$$
$$3(x - h)^2 + (y - k)^2 = 9$$
$$3(x + 1)^2 (y - 3)^2 = 9 \qquad (h, k) = (-1, 3)$$
$$3(x^2 + 2x + 1) + y^2 - 6y + 9 = 9$$
$$3x^2 + 6x + 3 + y^2 - 6y + 9 = 9$$
$$3x^2 + y^2 + 6x - 6y + 3 = 0$$

16. $$B^2 - 4AC = 0 - 4(12)(4) = -192$$
$A \neq C$; ellipse
$$4y^2 + 12x^2 = 24$$
$$4(y - k)^2 + 12(x - h)^2 = 24$$
$$4(y - 4)^2 + 12(x + 1)^2 = 24 \qquad (h, k) = (-1, 4)$$
$$4(y^2 - 8y + 16) + 12(x^2 + 2x + 1) = 24$$
$$4y^2 - 32y + 64 + 12x^2 + 24x + 12 = 24$$
$$y^2 - 8y + 16 + 3x^2 + 6x + 3 = 6$$
$$3x^2 + y^2 + 6x - 8y + 13 = 0$$

17. $$B^2 - 4AC = 0 - 4(9)(-25) = 900$$
hyperbola
$$9x^2 - 25y^2 = 225$$
$$9(x - h)^2 - 25(y - k)^2 = 225$$
$$9(x - 0)^2 - 25(y - 5)^2 = 225 \qquad (h, k) = (0, 5)$$
$$9x^2 - 25(y^2 - 10y + 25) = 225$$
$$9x^2 - 25y^2 + 250y - 850 = 0$$

18. $$(x - 3)^2 = 4y$$
$$x^2 + 6x + 9 - 4y = 0$$
$$B^2 - 4AC = 0 - 4(1)(0) = 0$$
parabola
$$(x + 3)^2 = 4y$$
$$(x + 3 - h)^2 = 4(y - k)$$
$$(x + 3 + 7)^2 = 4(y - 2) \qquad (h, k) = (-7, 2)$$
$$(x + 10)^2 = 4y - 8$$
$$x^2 + 20x + 100 = 4y - 8$$
$$x^2 + 20x - 4y + 108 = 0$$

19. $$B^2 - 4AC = 0 - 4(1)(0) = 0$$
parabola
$$x^2 - 8y = 0$$
$$(x' \cos 90° + y' \sin 90°)^2 - 8(-x' \sin 90° + y' \cos 90°) = 0$$
$$(y')^2 - 8(-x') = 0$$
$$(y')^2 + 8x = 0$$

20. $B^2 - 4AC = 0 - 4(2)(2)$
$= -16$
$A = C$; circle

$$
\begin{aligned}
2x^2 + 2y^2 &= 8 \\
2(x' \cos 30° + y' \sin 30°)^2 & \\
+ 2(-x' \sin 30° + y' \cos 30°)^2 &= 8 \\
2\left(\tfrac{\sqrt{3}}{2}x' + \tfrac{1}{2}y'\right)^2 + 2\left(-\tfrac{1}{2}x' + \tfrac{\sqrt{3}}{2}y'\right)^2 &= 8 \\
2\left[\tfrac{3}{4}(x')^2 + \tfrac{\sqrt{3}}{2}x'y' + \tfrac{1}{4}(y')^2\right] & \\
+ 2\left[\tfrac{1}{4}(x')^2 - \tfrac{\sqrt{3}}{2}x'y' + \tfrac{3}{4}(y')^2\right] &= 8 \\
\tfrac{3}{2}(x')^2 + \sqrt{3}x'y' + \tfrac{1}{2}(y')^2 + \tfrac{1}{2}(x')^2 & \\
- \sqrt{3}x'y' + \tfrac{3}{2}(y')^2 &= 8 \\
2(x')^2 + 2(y')^2 &= 8 \\
(x')^2 + (y')^2 - 4 &= 0
\end{aligned}
$$

21. $B^2 - 4AC = 0 - 4(1)(0)$
$= 0$
$A \neq C$; parabola

$$
\begin{aligned}
y^2 + 8x &= 0 \\
\left(-x' \sin \tfrac{\pi}{6} + y' \cos \tfrac{\pi}{6}\right)^2 + 8\left(x' \cos \tfrac{\pi}{6} + y' \sin \tfrac{\pi}{6}\right) &= 0 \\
\left(-\tfrac{1}{2}x' + \tfrac{\sqrt{3}}{2}y'\right)^2 + 8\left(\tfrac{\sqrt{3}}{2}x' + \tfrac{1}{2}y'\right) &= 0 \\
\tfrac{1}{4}(x')^2 - \tfrac{\sqrt{3}}{2}x'y' + \tfrac{3}{4}(y')^2 + 4\sqrt{3}x' + 4y' &= 0 \\
(x')^2 - 2\sqrt{3}x'y' + 3(y')^2 + 16\sqrt{3}x' + 16y' &= 0
\end{aligned}
$$

22. $B^2 - 4AC = 1^2 - 4(0)(0)$
$= 1$
hyperbola

$$
\begin{aligned}
xy &= -8 \\
\left(x' \cos \tfrac{\pi}{4} + y' \sin \tfrac{\pi}{4}\right)\left(-x' \sin \tfrac{\pi}{4} + y' \cos \tfrac{\pi}{4}\right) &= -8 \\
\left(\tfrac{\sqrt{2}}{2}x' + \tfrac{\sqrt{2}}{2}y'\right)\left(-\tfrac{\sqrt{2}}{2}x' + \tfrac{\sqrt{2}}{2}y'\right) &= -8 \\
-\tfrac{1}{2}(x')^2 + \tfrac{1}{2}x'y' - \tfrac{1}{2}x'y' + \tfrac{1}{2}(y')^2 &= -8 \\
(x')^2 - (y')^2 &= 16 \\
(x')^2 - (y')^2 - 16 &= 0
\end{aligned}
$$

23. $B^2 - 4AC = 0 - 4(1)(1)$
$= -4$
$A = C$; circle

$$
\begin{aligned}
x^2 - 5x + y^2 &= 3 \\
\left(x' \cos \tfrac{\pi}{3} + y' \sin \tfrac{\pi}{3}\right)^2 - 5\left(x' \cos \tfrac{\pi}{3} + y' \sin \tfrac{\pi}{3}\right) & \\
+ \left(-x' \sin \tfrac{\pi}{3} + y' \cos \tfrac{\pi}{3}\right)^2 &= 3 \\
\left(\tfrac{1}{2}x' + \tfrac{\sqrt{3}}{2}y'\right)^2 - 5\left(\tfrac{1}{2}x' + \tfrac{\sqrt{3}}{2}y'\right) & \\
+ \left(-\tfrac{\sqrt{3}}{2}x' + \tfrac{1}{2}y'\right)^2 &= 3 \\
\tfrac{1}{4}(x')^2 + \tfrac{\sqrt{3}}{2}x'y' + \tfrac{3}{4}(y')^2 - \tfrac{5}{2}x' - \tfrac{5\sqrt{3}}{2}y' & \\
+ \tfrac{3}{4}(x')^2 - \tfrac{\sqrt{3}}{2}x'y' + \tfrac{1}{4}(y')^2 &= 3 \\
(x')^2 + (y')^2 - \tfrac{5}{2}x' - \tfrac{5\sqrt{3}}{2}y &= 3 \\
2(x')^2 + 2(y')^2 - 5x' - 5\sqrt{3}y' - 6 &= 0
\end{aligned}
$$

24. $B^2 - 4AC = 0 - 4(16)(-4)$
$= 256$
hyperbola

$$
\begin{aligned}
16x^2 - 4y^2 &= 64 \\
16(x' \cos 60° + y' \sin 60°)^2 & \\
- 4(-x' \sin 60° + y' \cos 60°) &= 64 \\
16\left(\tfrac{1}{2}x' + \tfrac{\sqrt{3}}{2}y'\right)^2 - 4\left(-\tfrac{\sqrt{3}}{2}x' + \tfrac{1}{2}y'\right)^2 &= 64 \\
16\left[\tfrac{1}{4}(x')^2 + \tfrac{\sqrt{3}}{2}x'y' + \tfrac{3}{4}(y')^2\right] & \\
- 4\left[\tfrac{3}{4}(x')^2 - \tfrac{\sqrt{3}}{2}x'y' + \tfrac{1}{4}(y')^2\right] &= 64 \\
4(x')^2 + 8\sqrt{3}x'y' + 12(y')^2 & \\
- 3(x')^2 + 2\sqrt{3}x'y' - (y')^2 &= 64 \\
(x')^2 + 10\sqrt{3}x'y' + 11y')^2 - 64 &= 0
\end{aligned}
$$

25.

$$
\begin{aligned}
6x^2 + 5y^2 &= 30 \\
6(x' \cos 30° + y' \sin 30°)^2 & \\
+ 5(-x' \sin 30° + y' \cos 30°)^2 &= 30 \\
6\left(\tfrac{\sqrt{3}}{2}x' + \tfrac{1}{2}y'\right)^2 + 5\left(-\tfrac{1}{2}x' + \tfrac{\sqrt{3}}{2}y'\right)^2 &= 30 \\
6\left[\tfrac{3}{4}(x')^2 + \tfrac{\sqrt{3}}{2}x'y' + \tfrac{1}{4}(y')^2\right] & \\
+ 5\left[\tfrac{1}{4}(x')^2 - \tfrac{\sqrt{3}}{2}x'y' + \tfrac{3}{4}(y')^2\right] &= 30 \\
\tfrac{18}{4}(x')^2 + \tfrac{6\sqrt{3}}{2}x'y' + \tfrac{6}{4}(y')^2 + \tfrac{5}{4}(x')^2 - \tfrac{5\sqrt{3}}{2}x'y' & \\
+ \tfrac{15}{4}(y')^2 &= 30 \\
\tfrac{23}{4}(x')^2 + \tfrac{\sqrt{3}}{2}x'y' + \tfrac{21}{4}(y')^2 - 30 &= 0 \\
23(x')^2 + 2\sqrt{3}x'y' + 21(y')^2 - 120 &= 0
\end{aligned}
$$

26. $3^2 - 4AC = 4^2 - 4(9)(5)$
$= -164$
$A \neq C$; ellipse

$$
\begin{aligned}
\tan 2\theta &= \frac{B}{A - C} \\
\tan 2\theta &= \frac{4}{9 - 5} \\
\tan 2\theta &= 1 \\
2\theta &= 45° \\
\theta &\approx 23°
\end{aligned}
$$

27. $B^2 - 4AC = (-1)^2 - 4(1)(-4)$
$= 17$
hyperbola

$$
\begin{aligned}
\tan 2\theta &= \frac{B}{A - C} \\
\tan 2\theta &= \frac{-1}{1 - (-4)} \\
\tan 2\theta &= -\frac{1}{5} \\
2\theta &\approx -11.30993247° \\
\theta &\approx -6°
\end{aligned}
$$

28. $B^2 - 4AC = 8^2 - 4(8)(2)$
$= 0$
parabola

$$
\begin{aligned}
\tan 2\theta &= \frac{B}{A - C} \\
\tan 2\theta &= \frac{8}{8 - 2} \\
\tan 2\theta &= \frac{4}{3} \\
2\theta &\approx 53.13010235° \\
\theta &\approx 27°
\end{aligned}
$$

29. $B^2 - 4AC = 9^2 - 4(2)(14)$
$= -31$
$A \neq C$; ellipse

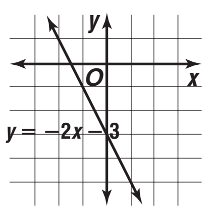
$$
\begin{aligned}
\tan 2\theta &= \frac{B}{A - C} \\
\tan 2\theta &= \frac{9}{2 - 14} \\
\tan 2\theta &= -\frac{3}{4} \\
2\theta &\approx -36.86989765° \\
\theta &\approx -18°
\end{aligned}
$$

30. $B^2 - 4AC = 4^2 - 4(2)(5)$
$= -24$
$A \neq C$; ellipse
$\tan 2\theta = \frac{B}{A - C}$
$\tan 2\theta = \frac{4}{2 - 5}$
$\tan 2\theta = -\frac{4}{3}$
$2\theta \approx -53.13010235°$
$\theta \approx -27°$

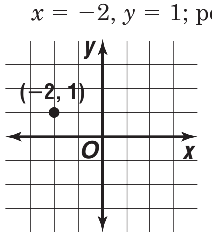
31. $B^2 - 4AC = \left(4\sqrt{3}\right)^2 - 4(2)(6)$
$= 0$
parabola
$\tan 2\theta = \frac{B}{A - C}$
$\tan 2\theta = \frac{4\sqrt{3}}{2 - 6}$
$\tan 2\theta = -\sqrt{3}$
$2\theta = -60°$
$\theta = -30°$

32. $B^2 - 4AC = 4^2 - 4(2)(2)$
$= 0$
parabola
$A = C$; $\theta = \frac{\pi}{4}$ or $45°$

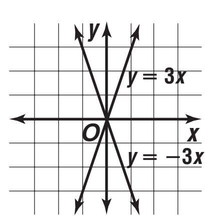
33.
$$\begin{aligned}(x - 2)^2 - (x + 3)^2 &= 5(y + 2)\\ x^2 - 4x + 4 - x^2 - 6x - 9 &= 5(y + 2)\\ -10x - 5 &= 5(y + 2)\\ -2x - 1 &= y + 2\\ -2x - 3 &= y\\ y &= -2x - 3 \quad \text{line}\end{aligned}$$



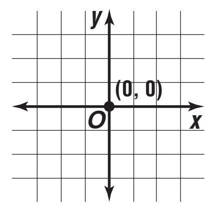
34. $2x^2 + 6y^2 + 8x - 12y + 14 = 0$
$x^2 + 3y^2 + 4x - 6y + 7 = 0$
$$\underset{\substack{\uparrow\\ a}}{3y^2} + \underset{\substack{\uparrow\\ b}}{(-6)y} + \underset{\substack{\uparrow\\ c}}{\underbrace{(x^2 + 4x + 7)}} = 0$$
$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$y = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(x^2 + 4x + 7)}}{2(3)}$
$y = \frac{6 \pm \sqrt{-12x^2 - 48x - 48}}{6}$
$y = \frac{6 \pm \sqrt{-12(x + 2)^2}}{6}$
$x = -2$, $y = 1$; point



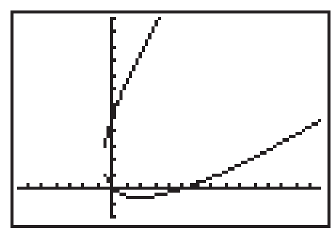
35. $y^2 - 9x^2 = 0$
$y^2 = 9x^2$
$y = \sqrt{9x^2}$
$y = \pm 3x$
intersecting lines



36.
$$\begin{aligned}(x - 2)^2 + (y - 2)^2 + 4(x + y) &= 8\\ x^2 - 4x + 4 + y^2 - 4y + 4 + 4x + 4y &= 8\\ \underset{\substack{\uparrow\\ a}}{(1)}y^2 + \underset{\substack{\uparrow\\ b}}{0}y + \underset{\substack{\uparrow\\ c}}{x^2} &= 0\end{aligned}$$
$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$y = \frac{0 \pm \sqrt{0 - 4(1)(x^2)}}{2(1)}$
$y = \pm \frac{\sqrt{-4x^2}}{2}$
$x = 0$, $y = 0$; point

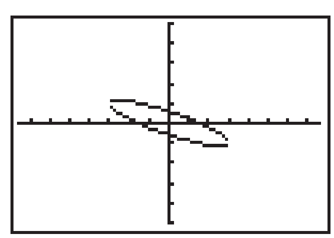


37. $x^2 - 2xy + y^2 - 5x - 5y = 0$
$$\underset{\substack{\uparrow\\ a}}{(1)}y^2 + \underset{\substack{\uparrow\\ b}}{\underbrace{(-2x - 5)}}y + \underset{\substack{\uparrow\\ c}}{\underbrace{(x^2 - 5x)}} = 0$$
$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$y = \frac{-(-2x - 5) \pm \sqrt{(-2x - 5)^2 - 4(1)(x^2 - 5x)}}{2(1)}$
$y = \frac{2x + 5 \pm \sqrt{4x^2 + 20x + 25 - 4x^2 + 20x}}{2}$
$y = \frac{2x + 5 \pm \sqrt{40x + 25}}{2}$



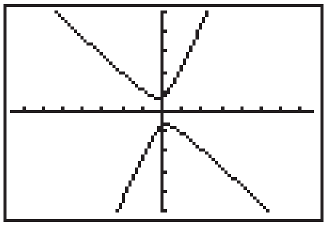
$[-6.61, 14.6]$ scl:1 by $[-2, 12]$ scl:1

38. $2x^2 + 9xy + 14y^2 = 5$
$$\underset{\substack{\uparrow\\ a}}{14y^2} + \underset{\substack{\uparrow\\ b}}{(9x)}y + \underset{\substack{\uparrow\\ c}}{\underbrace{(2x^2 - 5)}} = 0$$
$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$y = \frac{-9x \pm \sqrt{(9x)^2 - 4(14)(2x^2 - 5)}}{2(14)}$
$y = \frac{-9x \pm \sqrt{-21x^2 + 280}}{28}$



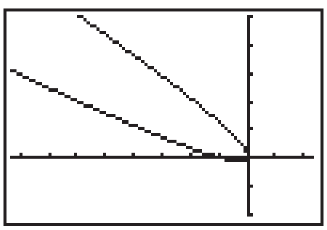
$[-7.58, 7.58]$ scl:1 by $[-5, 5]$ scl:1

39. $$8x^2 + 5xy - 4y^2 = -2$$
$$\underbrace{(-4)}_{a}y^2 + \underbrace{(5x)}_{b}y + \underbrace{(8x^2 + 2)}_{c} = 0$$
$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$y = \frac{-5x \pm \sqrt{(5x)^2 - 4(-4)(8x^2 + 2)}}{}$$
$$y = \frac{-5x \pm \sqrt{153x^2 + 32}}{}$$



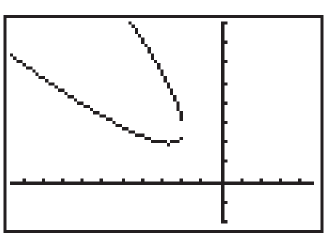
$[-7.58, 7.58]$ scl:1 by $[-5, 5]$ scl:1

40. $$2x^2 + 4\sqrt{3}xy + 6y^2 + 3x = y$$
$$\underbrace{6}_{a}y^2 + \underbrace{(4\sqrt{3}x - 1)}_{b}y + \underbrace{(2x^2 + 3x)}_{c} = 0$$
$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$y = \frac{-(4\sqrt{3}x - 1) \pm \sqrt{(4\sqrt{3}x - 1)^2 - 4(6)(2x^2 + 3x)}}{2(6)}$$
$$y = \frac{-4\sqrt{3}x + 1 \pm \sqrt{48x^2 - 8\sqrt{3}x + 1 - 48x^2 - 72x}}{12}$$
$$y = \frac{-4\sqrt{3}x + 1 \pm \sqrt{-8\sqrt{3}x - 72x + 1}}{12}$$



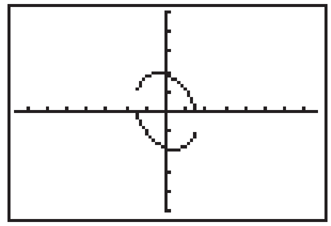
$[-8.31, 2.31]$ scl:1 by $[-2, 5]$ scl:1

41. $$2x^2 + 4xy + 2y^2 + 2\sqrt{2}x - 2\sqrt{2}y = -12$$
$$\underbrace{2}_{a}y^2 + \underbrace{(4x - 2\sqrt{2})}_{b}y + \underbrace{(2x^2 + 2\sqrt{2}x + 12)}_{c} = 0$$
$$y = \frac{-b \pm b^2 - 4ac}{2a}$$
$$y = \frac{-(4x - 2\sqrt{2}) \pm \sqrt{(4x - 2\sqrt{2})^2 - 4(2)(2x^2 + 2\sqrt{2}x + 12)}}{2(2)}$$
$$y = \frac{-4x + 2\sqrt{2} \pm \sqrt{16x^2 - 16\sqrt{2}x + 8 - 16x^2 - 16\sqrt{2}x - 96}}{4}$$
$$y = \frac{-4x + 2\sqrt{2} \pm \sqrt{-32\sqrt{2}x - 88}}{4}$$



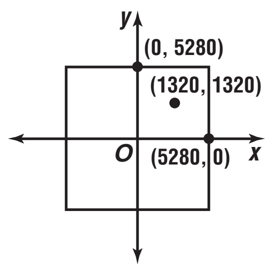
$[-10.58, 4.58]$ scl:1 by $[-2, 8]$ scl:1

42. $$9x^2 + 4xy + 6y^2 = 20$$
$$\underbrace{6}_{a}y^2 + \underbrace{(4x)}_{b}y + \underbrace{(9x^2 - 20)}_{c} = 0$$
$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$y = \frac{-4x \pm \sqrt{(4x)^2 - 4(6)(9x^2 - 20)}}{2(6)}$$
$$y = \frac{-4x \pm \sqrt{-212x^2 + 480}}{12}$$



$[-7.85, 7.85]$ scl:1 by $[-5, 5]$ scl:1

43a.



$T_{(1320,\ 1320)}$

43b. circle
center: $(h, k) = (1320, 1320)$
radius: $r = 1320$
$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 1320)^2 + (y - 1320)^2 = 1320^2$$
$$(x - 1320)^2 + (y - 1320)^2 = 1{,}742{,}400$$

44a. $B^2 - 4AC = 0 - 4(1)(0)$
$= 0$
parabola; 360°

44b. $B^2 - 4AC = 0 - 4(8)(6)$
$= -192$
$A \neq C$; ellipse; 180°

44c. $B^2 - 4AC = 4^2 - 4(0)(0)$
$= 16$
hyperbola; 180°

44d. $3^2 - 4AC = 0 - 4(15)(15)$
$= -900$
$A = C$; circle; There is no minimum angle of rotation, since any degree of rotation will result in a graph that coincides with the original.

45. Let $x = x' \cos \theta + y' \sin \theta$ and $y = -x' \sin \theta + y' \cos \theta$.
$$x^2 + y^2 = r^2$$
$$(x' \cos \theta + y' \sin \theta)^2 + (-x' \sin \theta + y' \cos \theta)^2 = r^2$$
$$(x')^2 \cos^2 \theta + x'y' \cos \theta \sin \theta + (y')^2 \sin^2 \theta$$
$$+ (x')^2 \sin^2 \theta - x'y' \cos \theta \sin \theta + (y')^2 \cos \theta = r^2$$
$$[(x')^2 + (y')^2] \cos^2 \theta + [(x')^2 + (y')^2] \sin^2 \theta = r^2$$
$$[(x')^2 + (y')^2](\cos^2 \theta + \sin^2 \theta) = r^2$$
$$[(x')^2 + (y')^2](1) = r^2$$
$$(x')^2 + (y')^2 = r^2$$

46a. $B^2 - 4AC = \left(-10\sqrt{3}\right)^2 - 4(31)(21)$
$= -2304$
$A \neq C$; elliptical

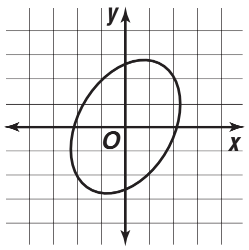
46b. $31x^2 - 10\sqrt{3}xy + 21y^2 = 144$

$$\underset{a}{\underset{\uparrow}{21y^2}} + \underset{b}{\underset{\uparrow}{(-10\sqrt{3}x)y}} + \underset{c}{\underset{\uparrow}{\underbrace{(31x^2 - 144)}}} = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-10\sqrt{3}x) \pm \sqrt{(-10\sqrt{3}x)^2 - 4(21)(31x^2 - 144)}}{2(21)}$$

$$y = \frac{10\sqrt{3}x \pm \sqrt{-2304x^2 + 12{,}096}}{42}$$



46c.

$$\tan 2\theta = \frac{B}{A - C}$$

$$\tan 2\theta = \frac{-10\sqrt{3}}{31 - 21}$$

$$\tan 2\theta = -\sqrt{3}$$

$$2\theta = -60°$$

$$\theta = -30°$$

47a.

$$\tan 2\theta = \frac{B}{A - C}$$

$$\tan 2\theta = \frac{-2\sqrt{3}}{9 - 11}$$

$$\tan 2\theta = \sqrt{3}$$

$$2\theta = 60°$$

$$\theta = 30°$$

The graph of this equation has been rotated 30°. To transform the graph so the axes are on the x- and y-axes, rotate the graph $-30°$.

47b. $B^2 - 4AC = \left(-2\sqrt{3}\right)^2 - 4(9)(11)$

$= -384$

$A \neq C$; the graph is an ellipse.

$$9x^2 - 2\sqrt{3}xy + 11y^2 - 24 = 0$$

$$9[x' \cos(-30°) + y' \sin(-30°)]^2 - 2\sqrt{3}$$
$$[(x' \cos(-30°) + y' \sin(-30°)]$$
$$[-x' \sin(-30°) + y' \cos(-30°)] + 11$$
$$[-x' \sin(-30°) + y' \cos(-30°)^2] - 24 = 0$$

$$9\left(\frac{\sqrt{3}}{2}x' - \frac{1}{2}y'\right)^2 - 2\sqrt{3}\left(\frac{\sqrt{3}}{2}x' - \frac{1}{2}y'\right)$$
$$\left(\frac{1}{2}x' + \frac{\sqrt{3}}{2}y'\right) + 11\left(\frac{1}{2}x' + \frac{\sqrt{3}}{2}y'\right)^2 - 24 = 0$$

$$9\left[\frac{3}{4}(x')^2 - \frac{\sqrt{3}}{2}x'y' + \frac{1}{4}(y')^2\right] -$$
$$2\sqrt{3}\left[\frac{\sqrt{3}}{4}(x')^2 + \frac{3}{4}x'y' - \frac{1}{4}x'y' - \frac{\sqrt{3}}{4}(y')^2\right]$$
$$+ 11\left[\frac{1}{4}(x')^2 + \frac{\sqrt{3}}{2}x'y' + \frac{3}{4}(y')^2\right] - 24 = 0$$

$$\frac{27}{4}(x')^2 - \frac{9\sqrt{3}}{2}x'y' + \frac{9}{4}(y')^2 - \frac{6}{4}(x')^2$$
$$- \frac{6\sqrt{3}}{4}x'y' + \frac{2\sqrt{3}}{4}x'y' + \frac{6}{4}(y')^2$$
$$+ \frac{11}{4}(x')^2 + \frac{11\sqrt{3}}{2}x'y' + \frac{33}{4}(y')^2 - 24 = 0$$

$$8(x')^2 + 12(y')^2 - 24 = 0$$

$$\frac{(x')^2}{3} + \frac{(y')^2}{2} = 1$$

48a. center: $(h, k) = (0, 0)$

major axis: horizontal

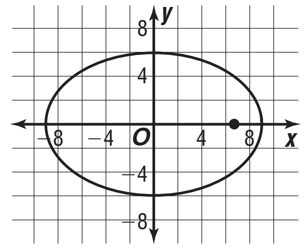
$a = \sqrt{81}$ or 9

$b = \sqrt{36}$ or 6

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{81 - 36}$

$c = \sqrt{45}$ or $3\sqrt{5}$



48b. $T_{(3\sqrt{5},\, 0)}$

$$\frac{x^2}{81} + \frac{y^2}{36} = 1$$

$$\frac{(x - h)^2}{81} + \frac{(y - k)^2}{36} = 1$$

48c. $\dfrac{(x' - 3\sqrt{5})^2}{36} + \dfrac{(y')^2}{81} = 1$

49. $A = -3$, $C = 5$; since A and C have different signs, the conic is a hyperbola.

50. $(h, k) = (2, -3)$

$$e = \frac{c}{a}$$

$$\frac{2\sqrt{6}}{5} = \frac{c}{1}$$

$$\frac{2\sqrt{6}}{5} = c$$

$$b^2 = a^2 - c^2$$

$$b^2 = 1^2 - \left(\frac{2\sqrt{6}}{5}\right)^2$$

$$b^2 = \frac{1}{25}$$

major axis: horizontal

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

$$\frac{(x - 2)^2}{1} + \frac{[y - (-3)]^2}{\frac{1}{25}} = 1$$

$$(x - 2)^2 + 25(y + 3)^2 = 1$$

major axis : vertical

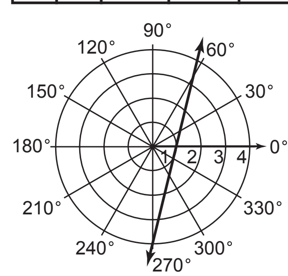
$$\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1$$

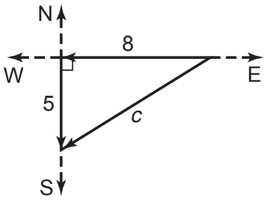
$$\frac{[y - (-3)]^2}{1^2} + \frac{(x - 2)^2}{\frac{1}{25}} = 1$$

$$(y + 3)^2 + 25(x - 2)^2 = 1$$

51.

θ	0	30°	60°	90°	120°	150°	180°
r	1	1.4	3.9	−3.9	−1.4	−1	−1



52. 

$c^2 = a^2 + b^2$
$c^2 = 8^2 + 5^2$
$c = \sqrt{89}$
$c \approx 9.4$
about 9.4 m/s

53. $\cos 70° \approx 0.34$
$\cos 170° \approx -0.98$
$\cos 70°$

54. $\frac{5\pi}{16} = \frac{5\pi}{16} \times \frac{180°}{\pi}$
$= 56.25$
$= 56\frac{15}{60}$
$= 56° \, 15'$

55. $\frac{2y + 5}{y^2 + 3y + 2} = \frac{2y + 5}{(y + 2)(y + 1)}$
$\frac{2y + 5}{y^2 + 3y + 2} = \frac{A}{y + 2} + \frac{B}{y + 1}$
$2y + 5 = A(y + 1) + B(y + 2)$
$2(-2) + 5 = A(-2 + 1) + B(-2 + 2)$
$1 = -A$
$-1 = A$
$2y + 5 = A(y + 1) + B(y + 2)$
$2(-1) + 5 = A(-1 + 1) + B(-1 + 2)$
$3 = B$
$\frac{A}{y + 2} + \frac{B}{y + 1} = \frac{-1}{y + 2} + \frac{3}{y + 1}$

56. $\frac{x_1}{y_2} = \frac{x_2}{y_1}$
$\frac{12}{y_2} = \frac{5}{4}$
$y_2 = 9.6$

57. $8m - 3n - 4p = 6$
$8m = 6 + 3n + 4p$
$m = \frac{3}{4} + \frac{3}{8}n + \frac{1}{2}p$
$4m + 9n - 2p = -4$
$4\left(\frac{3}{4} + \frac{3}{8}n + \frac{1}{2}p\right) + 9n - 2p = -4$
$3 + \frac{3}{2}n + 2p + 9n - 2p = -4$
$\frac{21}{2}n = -7$
$n = -\frac{2}{3}$

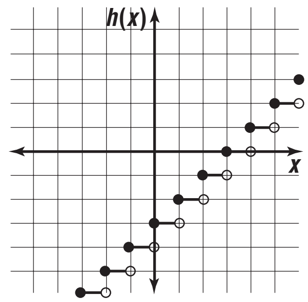
$8m - 3n - 4p = 6$ $\quad$ $6m + 12n + 5p = -1$
$8m - 3\left(-\frac{2}{3}\right) - 4p = 6$ $\quad$ $6m + 12\left(-\frac{2}{3}\right) + 5p = -1$
$8m + 2 - 4p = 6$ $\quad$ $6m - 8 + 5p = -1$
$8m - 4p = 4$ $\quad$ $6m + 5p = 7$
$2m - p = 1$

$2m - p = 1 \rightarrow 10m - 5p = 5$
$6m + 5p = 7 \rightarrow (+)\ 6m + 5p = 7$
$16m = 12$
$m = \frac{3}{4}$

$8m - 3n - 4p = 6$
$8\left(\frac{3}{4}\right) - 3\left(-\frac{2}{3}\right) - 4p = 6$
$6 + 2 - 4p = 6$
$p = \frac{1}{2}$
$\left(\frac{3}{4}, -\frac{2}{3}, \frac{1}{2}\right)$

58.

x	$h(x) = [[x]] - 3$
$-3 \le x < -2$	-6
$-2 \le x < -1$	-5
$-1 \le x < 0$	-4
$0 \le x < 1$	-3
$1 \le x < 2$	-2
$2 \le x < 3$	-1
$3 \le x < 4$	0
$4 \le x < 5$	1
$5 \le x < 6$	2
$6 \le x < 7$	3

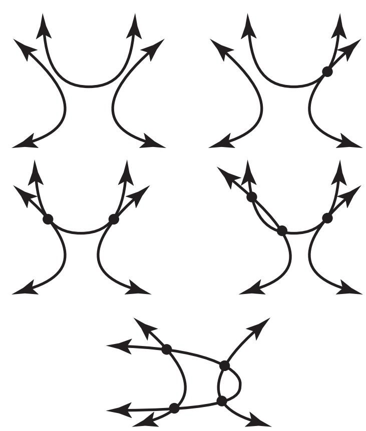


59. The expression $\frac{5a^8b^5}{180a^6b^2}$ simplifies to $\frac{a^2b^3}{36}$. Since $1 < b$ and $2 < a$, the expression is always larger than $\frac{2^2 1^3}{36} = \frac{1}{9}$. Since $b < 2$ and $a < 3$, the expression is always less than $\frac{3^2 2^3}{36} = \frac{72}{36}$ or 2. The correct choice is B.

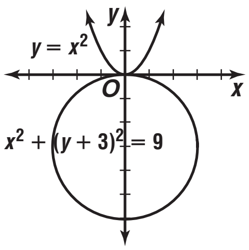
10-8 Systems of Second-Degree Equations and Inequalities

Page 682 Check for Understanding

1. Possible number of solutions: 0, 1, 2, 3, or 4



2. Sample answer: $y = x^2$, $x^2 + (y + 3)^2 = 9$



3. The system contains equation(s) that are equivalent. The graphs coincide.

4. Graph each second-degree inequality. The region in which the graphs overlap represents the solution to the system.

5. $x - y = 0$

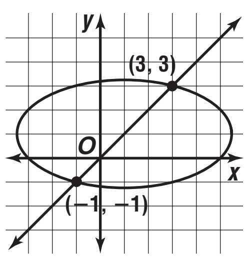
$x = y$

$$\frac{(x-1)^2}{20} + \frac{(y-1)^2}{5} = 1$$
$$\frac{(x-1)^2}{20} + \frac{(x-1)^2}{5} = 1$$
$$(x-1)^2 + 4(x-1)^2 = 20$$
$$x^2 - 2x + 1 + 4(x^2 - 2x + 1) = 20$$
$$5x^2 - 10x + 5 - 20 = 0$$
$$5(x^2 - 2x - 3) = 0$$
$$5(x - 3)(x + 1) = 0$$

$x - 3 = 0$ $\quad$ $x + 1 = 0$

$x = 3$ $\quad$ $x = -1$

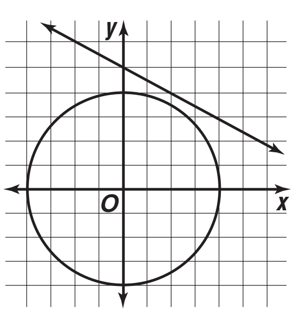
(3, 3), (−1, −1)



6. $x + 2y = 10$

$$x = 10 - 2y$$
$$x^2 + y^2 = 16$$
$$(10 - 2y)^2 + y^2 = 16$$
$$100 - 40y + 5y^2 = 16$$
$$5y^2 - 40y + 84 = 0$$
$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$y = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(5)(84)}}{2(5)}$$
$$y = \frac{40 \pm \sqrt{-80}}{10}$$

no solution

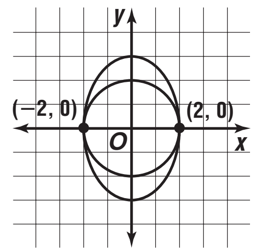


7. $x^2 + y^2 = 4$

$$y^2 = 4 - x^2$$
$$9x^2 - 4y^2 = 36$$
$$9x^2 - 4(4 - x^2) = 36$$
$$13x^2 - 16 = 36$$
$$x^2 = 4$$
$$x = \pm 2$$

$$x^2 + y^2 = 4$$
$$2^2 + y^2 = 4$$
$$y^2 = 0$$
$$y = 0$$

(±2, 0)

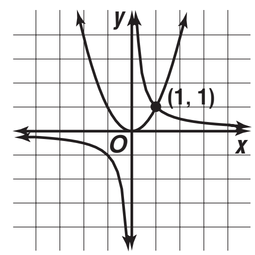


8. $xy = 1$

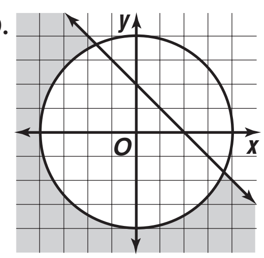
$$x(x^2) = 1$$
$$x^3 = 1$$
$$x = 1$$

$$x^2 = y$$
$$1^2 = y$$
$$1 = y$$

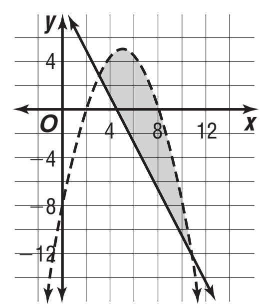
(1, 1)



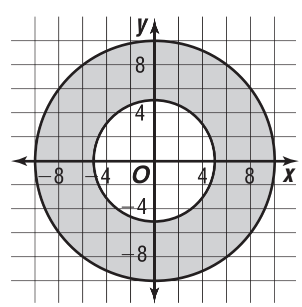
9.



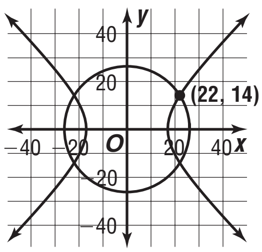
10.



11.



12a. Let x = side length of flowerbed 1.
Let y = side length of flowerbed 2.
$A_1 = x \cdot x$ or x^2
$A_2 = y \cdot y$ or y^2
Total Area $= x^2 + y^2$
$680 = x^2 + y^2$
$x^2 + y^2 = 680$
Difference of Areas $= x^2 - y^2$
$288 = x^2 - y^2$
$x^2 - y^2 = 288$

12b.


Since side length cannot be negative, an estimated solution is (22, 14).

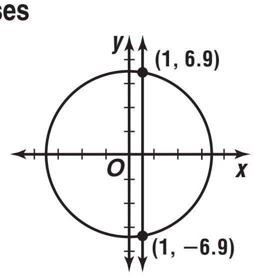
12c. $x^2 + y^2 = 680$
$y^2 = 680 - x^2$

$x^2 - y^2 = 288$	$x^2 + y^2 = 680$
$x^2 - (680 - x^2) = 288$	$22^2 + y^2 = 680$
$2x^2 = 968$	$y^2 = 196$
$x^2 = 484$	$y = 14$
$x = 22$	

22 ft and 14 ft

Pages 682–684 Exercises

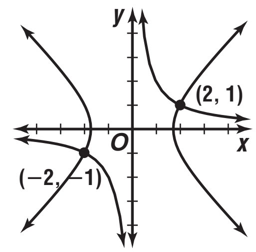
13. $x - 1 = 0$
$x = 1$
$y^2 = 49 - x^2$
$y^2 = 49 - (-1)^2$
$y^2 = 48$
$y \approx \pm 6.9$
$(1, \pm 6.9)$



14. $xy = 2$
$y = \frac{2}{x}$
$x^2 = 3 + y^2$
$x^2 = 3 + \left(\frac{2}{x}\right)^2$
$x^2 = 3 + \frac{4}{x^2}$
$x^4 = 3x^2 + 4$
$x^4 - 3x^2 - 4 = 0$
$(x^2 - 4)(x^2 + 1) = 0$

$x^2 - 4 = 0$	$x^2 + 1 = 0$
$x^2 = 4$	$x^2 = -1$
$x = \pm 2$	
$xy = 2$	$xy = 2$
$2(y) = 2$	$(-2)y = 2$
$y = 1$	$y = -1$

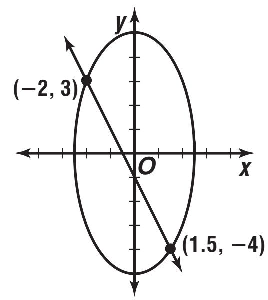
(2, 1), (−2, −1)



15. $-1 = 2x + y$
$-1 - 2x = y$
$4x^2 + y^2 = 25$
$4x^2 + (-1 - 2x)^2 = 25$
$4x^2 + 1 + 4x + 4x^2 = 25$
$8x^2 + 4x - 24 = 0$
$4(2x^2 + x - 6) = 0$
$4(2x - 3)(x + 2) = 0$

$2x - 3 = 0$	$x + 2 = 0$
$x = 1.5$	$x = -2$
$-1 = 2x + y$	$-1 = 2x + y$
$-1 = 2(1.5) + y$	$-1 = 2(-2) + y$
$-4 = y$	$3 = y$

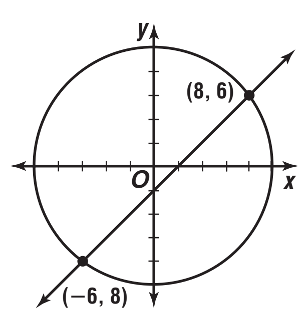
(1.5, −4), (−2, 3)



16. $x - y = 2$
$x = 2 + y$
$x^2 = 100 - y^2$
$(2 + y)^2 = 100 - y^2$
$4 + 4y + y^2 = 100 - y^2$
$2y^2 + 4y - 96 = 0$
$2(y^2 + 2y - 48) = 0$
$2(y - 6)(y + 8) = 0$

$y - 6 = 0$	$y + 8 = 0$
$y = 6$	$y = -8$
$x - y = 2$	$x - y = 2$
$x - 6 = 2$	$x - (-8) = 2$
$x = 8$	$x = -6$

(8, 6), (−6, −8)



17. $x - y = 0$

$x = y$

$\frac{(x-1)^2}{9} - y^2 = 1$

$\frac{(y-1)^2}{9} - y^2 = 1$

$(y-1)^2 - 9y^2 = 9$

$y^2 - 2y + 1 - 9y^2 = 9$

$-8y^2 - 2y - 8 = 0$

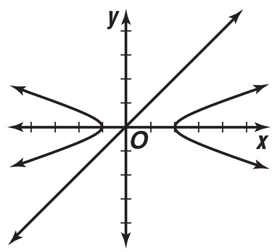
$4y^2 + y + 4 = 0$

$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \frac{-1 \pm \sqrt{1^2 - 4(4)(4)}}{2(1)}$

$y = \frac{-1 \pm \sqrt{-63}}{2}$

no solution



18. $x^2 + 2y^2 = 10$

$x^2 = 10 - 2y^2$

$3x^2 = 9 - y^2$

$3(10 - 2y^2) = 9 - y^2$

$30 - 6y^2 = 9 - y^2$

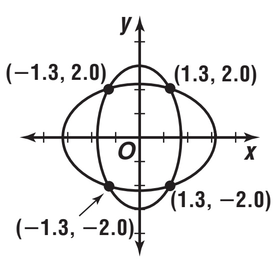
$-5y^2 = -21$

$y^2 = 4.2$

$y \approx \pm 2.0$

$3x^2 = 9 - y^2$	$3x^2 = 9 - y^2$
$3x^2 \approx 9 - (2.0)^2$	$3x^2 \approx 9 - (-2.0)^2$
$x^2 \approx 1.6$	$x^2 \approx 1.6$
$x \approx \pm 1.3$	$x = \pm 1.3$

$(\pm 1.3, 2.0), (\pm 1.3, -2.0)$



19. $x + y = -1$

$y = -1 - x$

$(y-1)^2 = 4 + x$

$(-1 - x - 1)^2 = 4 + x$

$(-2 - x)^2 = 4 + x$

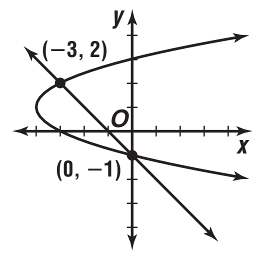
$4 + 4x + x^2 = 4 + x$

$x^2 + 3x = 0$

$x(x + 3) = 0$

$x = 0$	$x + 3 = 0$
	$x = -3$
$x + y = -1$	$x + y = -1$
$0 + y = -1$	$-3 + y = -1$
$y = -1$	$y = 2$

$(0, -1), (-3, 2)$



20. $xy + 6 = 0$

$y = -\frac{6}{x}$

$x^2 + y^2 = 13$

$x^2 + \left(-\frac{6}{x}\right)^2 = 13$

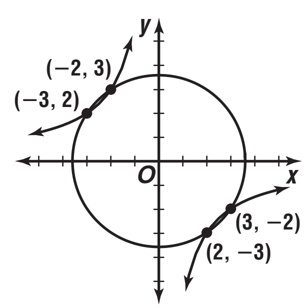
$x^2 + \frac{36}{x^2} = 13$

$x^4 + 36 = 13x^2$

$x^4 - 13x^2 + 36 = 0$

$(x^2 - 9)(x^2 - 4) = 0$

$x^2 - 9 = 0$	$x^2 - 4 = 0$
$x^2 = 9$	$x^2 = 4$
$x = \pm 3$	$x = \pm 2$
$xy + 6 = 0$	$xy + 6 = 0$
$3y + 6 = 0$	$2y + 6 = 0$
$y = -2$	$y = -3$
$xy + 6 = 0$	$xy + 6 = 0$
$-3y + 6 = 0$	$-2y + 6 = 0$
$y = 2$	$y = 3$
$(3, -2), (-3, 2)$	$(2, -3), (-2, 3)$



21. $x^2 + y - 3 = 0$

$x^2 = -y + 3$

$x^2 + 4y^2 = 36$

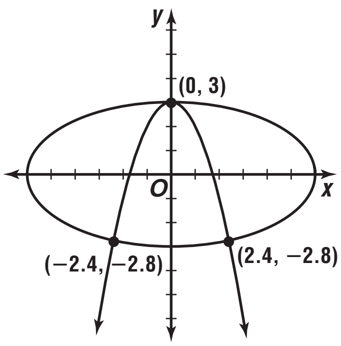
$-y + 3 + 4y^2 = 36$

$4y^2 - y - 33 = 0$

$(y - 3)(4y + 11) = 0$

$y - 3 = 0$	$4y + 11 = 0$
$y = 3$	$y \approx -2.8$
$x^2 + y - 3 = 0$	$x^2 + y - 3 = 0$
$x^2 + 3 - 3 = 0$	$x^2 + (-2.8) - 3 \approx 0$
$x^2 = 0$	$x^2 \approx 5.8$
$x = 0$	$x \approx \pm 2.4$

$(0, 3), (\pm 2.4, -2.8)$



22. $2y - x + 3 = 0$

$2y + 3 = x$

$x^2 = 16 - y^2$

$(2y + 3)^2 = 16 - y^2$

$4y^2 + 12y + 9 = 16 - y^2$

$5y^2 + 12y - 7 = 0$

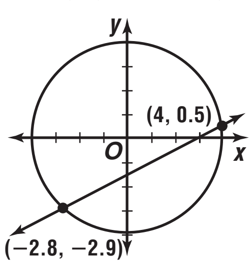
$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \frac{-12 \pm \sqrt{12^2 - 4(5)(-7)}}{2(5)}$

$y = \frac{-12 \pm \sqrt{284}}{10}$

$y \approx 0.5$	or	$y \approx -2.9$
$2y - x + 3 = 0$		$2y - x + 3 = 0$
$2(0.5) - x + 3 \approx 0$		$2(-2.9) - x + 3 \approx 0$
$x \approx 4.0$		$x \approx -2.8$

$(4.0, 0.5), (-2.8, -2.9)$



23. $xy = -4$

$y = -\frac{4}{x}$

$x^2 = 25 - 9y^2$

$x^2 = 25 - 9\left(-\frac{4}{x}\right)^2$

$x^2 = 25 - \frac{144}{x^2}$

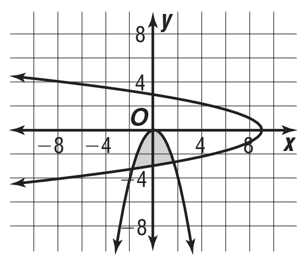
$x^4 = 25x^2 - 144$

$x^4 - 25x^2 + 144 = 0$

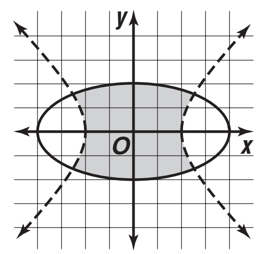
$(x^2 - 9)(x^2 - 16) = 0$

$x^2 - 9 = 0$	$x^2 - 16 = 0$
$x^2 = 9$	$x^2 = 16$
$x = \pm 3$	$x = \pm 4$
$xy = -4$	$xy = -4$
$3y = -4$	$4y = -4$
$y \approx -1.3$	$y = -1$
$xy = -4$	$xy = -4$
$-3y = -4$	$-4y = -4$
$y \approx 1.3$	$y = 1$
$(3, -1.3), (-3, 1.3)$	$(4, -1), (-4, 1)$

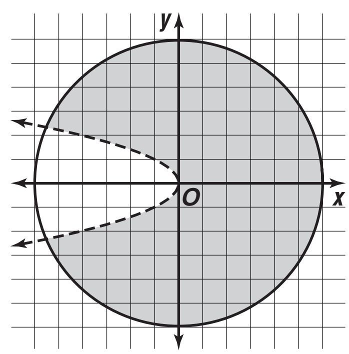
24.



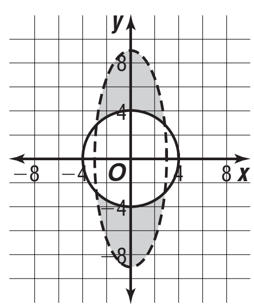
25.



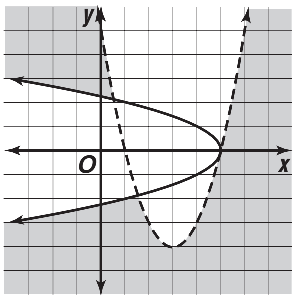
26.



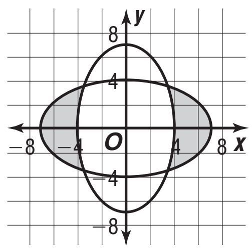
27.



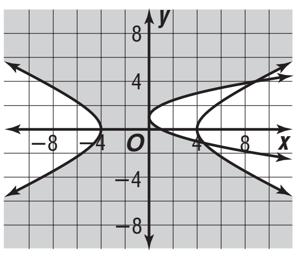
28.



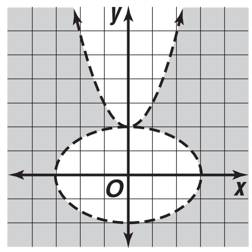
29.



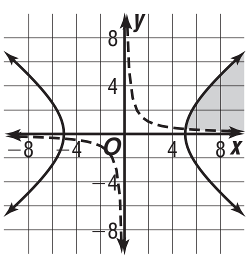
30.



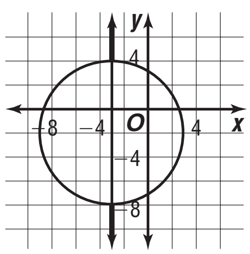
31.



32.



33.



34. parabola:

vertex: $(1, -3)$

$(y - k)^2 = 4p(x - h)$

$(-5 + 3)^2 = 4p(-1 - 1)$

$-\frac{1}{2} = p$

$(y - 3)^2 = 4\left(-\frac{1}{2}\right)(x - 1)$

$(y + 3)^2 = -2(x - 1)$

line:

$m = -2, b = -7$

$y = mx + b$

$y = -2x - 7$

35. circle:

center: $(0, 0)$, radius: $2\sqrt{2}$

$x^2 + y^2 = r^2$

$x^2 + y^2 = 8$

hyperbola:

$(-2)(-2) = 4$

$xy = 4$

36. large ellipse:

$a = 5, b = 4$, center $= (0, 0)$

$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$

$\frac{y^2}{25} + \frac{x^2}{16} \leq 1$ (interior is shaded)

small ellipse:

$a = 3, b = 2$, center $= (0, -1)$

$\frac{x^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$

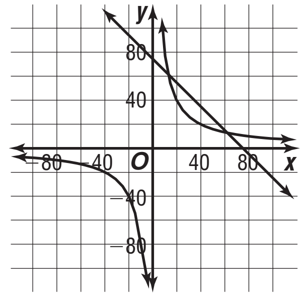
$\frac{x^2}{9} + \frac{(y + 1)^2}{4} > 1$ (exterior is shaded)

37a. $2x + 2y = P$ $\quad xy = A$

$2x + 2y = 150$ $\quad xy = 800$

37b. A system of a line and a hyperbola may have 0, 1, or 2 solutions.

37c.



37d. $xy = 800$

$y = \frac{800}{x}$

$2x + 2y = 150$

$2x + 2\left(\frac{800}{x}\right) = 150$

$2x + \frac{1600}{x} = 150$

$2x^2 + 1600 = 150x$

$x^2 - 75x + 800 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-75) \pm \sqrt{(-75)^2 - 4(1)(800)}}{2(1)}$

$x = \frac{75 \pm \sqrt{2425}}{2}$

$x \approx 12.88$ or $x \approx 62.12$

$xy = 800$ $\quad xy = 800$

$12.88y \approx 800$ $\quad 62.12y \approx 800$

$y \approx 62.11$ $\quad y \approx 12.88$

12.9 m by 62.1 m or 62.1 m by 12.9 m

38a. $(h, k) = (0, 4)$

$(x, y) = (6, 0)$

$(x - h)^2 = 4p(y - k)$

$(6 - 0)^2 = 4p\ (0, 4)$

$36 = -16p$

$-2.25 = p$

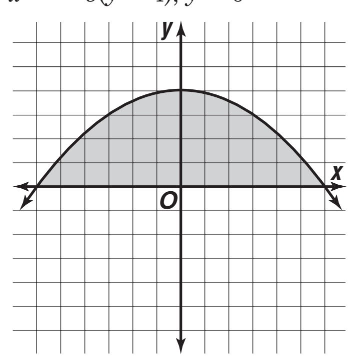
$(x - h)^2 = 4p(y - k)$

$(x - 0)^2 = 4(-2.25)(y - 4)$

$x^2 = -9(y - 4)$

$x^2 \leq -9(y - 4), y \geq 0$

38b.



38c. $(h, k) = (0, 3)$
$(x, y) = (6, 0)$
$(x - h)^2 = 4p(y - k)$
$(6 - 0)^2 = 4p(0 - 3)$
$36 = -12p$
$-3 = p$
$(x - h)^2 = 4p(y - k)$
$(x - 0)^2 = 4(-3)(y - 3)$
$x^2 = -12(y - 3)$
$x^2 \leq -12(y - 3),\ y \geq 0$

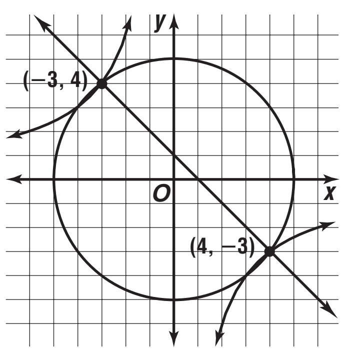
39. $xy = -12$
$y = -\frac{12}{x}$
$x = -y + 1$
$x = -\left(-\frac{12}{x}\right) + 1$
$x^2 = 12 + x$
$x^2 - x - 12 = 0$
$(x - 4)(x + 3) = 0$

$x - 4 = 0$	$x + 3 = 0$
$x = 4$	$x = -3$
$xy = -12$	$xy = -12$
$4y = -12$	$-3y = -12$
$y = -3$	$y = 4$
$(4, -3)$	$(-3, 4)$

Check that $(4, -3)$ and $(-3, 4)$ are also solutions of $y^2 = 25 - x^2$.

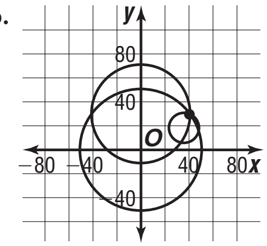
$y^2 = 25 - x^2$	$y^2 = 25 - x^2$
$(-3)^2 \stackrel{?}{=} 25 - (4)^2$	$(4)^2 \stackrel{?}{=} 25 - (-3)^2$
$9 = 9$ ✓	$16 = 16$ ✓

$(4, -3), (-3, 4)$



40a. first station: $(h, k = (0, 0)$
$x^2 + y^2 = r^2$
$x^2 + y^2 = 50^2$
$x^2 + y^2 = 2500$
second station: $(h, k) = (0, 30)$
$(x - h)^2 + (y - k)^2 = r^2$
$x^2 + (y - 30)^2 = 40^2$
$x^2 + (y - 30)^2 = 1600$
third station: $(h, k) = (35, 18)$
$(x - h)^2 + (y - k)^2 = r^2$
$(x - 35)^2 + (y - 18)^2 = 13^2$
$(x - 35)^2 + (y - 18)^2 = 169$

40b.



estimate: (40, 30)

40c. $x^2 + y^2 = 2500$
$x^2 = 2500 - y^2$
$x^2 + (y - 30)^2 = 1600$
$2500 - y^2 + y^2 - 60y + 900 = 1600$
$-60y + 1800 = 0$
$y = 30$
$(x - 35)^2 + (y - 18)^2 = 169$
$x^2 - 70x + 1225 + (30 - 18)^2 = 169$
$x^2 - 70x + 1200 = 0$
$(x - 30)(x - 40) = 0$

$x - 30 = 0$	or	$x - 40 = 0$
$x = 30$		$x = 40$

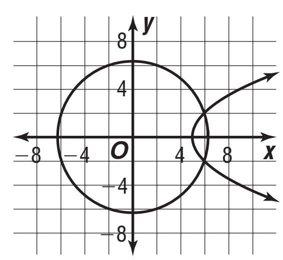
Check (30, 30) and (40, 30):

$x^2 + y^2 = 2500$	$x^2 + y^2 = 2500$
$30^2 + 30^2 \stackrel{?}{=} 2500$	$40^2 + 30^2 \stackrel{?}{=} 2500$
$1800 \neq 2500$	$2500 = 2500$ ✓

(40, 30)

41. $x + 3y = k$
$2y^2 + 3y = k$
$2y^2 + 3y - k = 0$
$y^2 + \frac{3}{2}y - \frac{1}{2}k = 0$
$y^2 + \frac{3}{2}y + \left(\frac{3}{4}\right)^2 = 0$ *Complete the square.*
$\left(y + \frac{3}{4}\right)^2 = 0$
$-\frac{1}{2}k = \left(\frac{3}{4}\right)^2$
$-\frac{1}{2}k = \frac{9}{16}$
$k = -\frac{9}{8}$

42a.



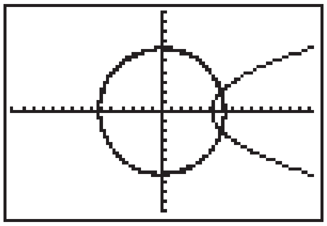
42b. yes; (6, 2) or (6, −2)

42c. Earth's surface:
$x_1{}^2 + y_1{}^2 = 40$
$\frac{x_1{}^2}{40} + \frac{y_1{}^2}{40} = 1$
$\left(\frac{x_1}{\sqrt{40}}\right)^2 + \left(\frac{y_1}{\sqrt{40}}\right)^2 = 1$
$\cos^2 t + \sin^2 t = 1$

$\left(\frac{x_1}{\sqrt{40}}\right)^2 = \cos^2 t$	$\left(\frac{y_1}{\sqrt{40}}\right)^2 = \sin^2 t$
$\frac{x_1}{2\sqrt{10}} = \cos t$	$\frac{y_1}{2\sqrt{10}} = \sin t$
$x_1 = 2\sqrt{10}\cos t$	$y_1 = 2\sqrt{10}\sin t$

asteroid:
Let $y_2 = -t$.
$x_2 = 0.25y^2 + 5$
$x_2 = 0.25\,t^2 + 5$

42d.



Tmin = −8, Tmax = 8, Tstep = 0.13
[−15.16, 15.16] scl:1 by [−10, 10] scl = 1

43.

$$\frac{x^2}{9} + y^2 = 1$$

$$\frac{(x'\cos 30° + y'\sin 30°)^2}{9} + (-x'\sin 30° + y'\cos 30°)^2 = 1$$

$$\frac{\left(\frac{\sqrt{3}}{2}x' + \frac{1}{2}y'\right)^2}{9} + \left(-\frac{1}{2}x' + \frac{\sqrt{3}}{2}y'\right)^2 = 1$$

$$\frac{\frac{3}{4}(x')^2 + \frac{\sqrt{3}}{2}x'y' + \frac{1}{4}(y')^2}{9} + \frac{1}{4}(x')^2 - \frac{\sqrt{3}}{2}x'y' + \frac{3}{4}(y')^2 = 1$$

$$\frac{3}{4}(x')^2 + \frac{\sqrt{3}}{2}x'y' + \frac{1}{4}(y')^2 + \frac{9}{4}(x')^2 - \frac{9\sqrt{3}}{2}x'y' + \frac{27}{4}(y')^2 = 9$$

$$3(x')^2 - 4\sqrt{3}x'y' + 7(y')^2 - 9 = 0$$

44. $x = 4t + 1$ $\qquad y = 5t - 7$

$\frac{x-1}{4} = t$ $\qquad \frac{y+7}{5} = t$

$$\frac{x-1}{4} = \frac{y+7}{5}$$

$(x - 1)(5) = (4)(y + 7)$
$5x - 5 = 4y + 28$
$5x - 4y - 33 = 0$

45. $4\csc\theta\cos\theta\tan\theta = 4\left(\frac{1}{\sin\theta}\right)(\cos\theta)\left(\frac{\sin\theta}{\cos\theta}\right)$
$= 4$

46. $r = 10$ cm or 0.10 m
$v = r\omega$
$v = (0.10)\left(\frac{5 \cdot 2\pi}{1}\right)$
$v \approx 3.14$ m/s

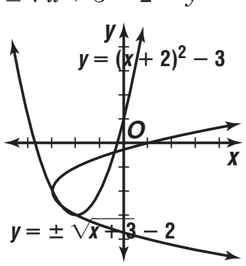
47.

r	1	0	0	−4
0	1	0	0	−4
1	1	1	1	−3
2	1	2	4	4

between 1 and 2

48.

$y = (x + 2)^2 - 3$
$x = (y + 2)^2 - 3$
$x + 3 = (y + 2)^2$
$\pm\sqrt{x + 3} = y + 2$
$\pm\sqrt{x + 3} - 2 = y$



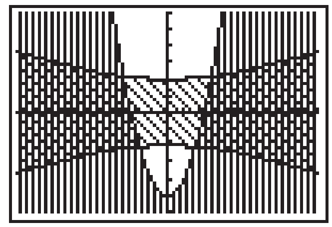
49. No; the domain value 4 is mapped to two elements in the range, 0 and −3.

50. area of rectangle $= \ell\omega$
$= 8(4)$ or 32
area of circles $= 2\,(\pi r^2)$
$= 2(4\pi)$ or 8π
area of shaded region $= 32 - 8\pi$
The correct choice is E.

10-8B Graphing Calculator Exploration: Shading Areas on a Graph

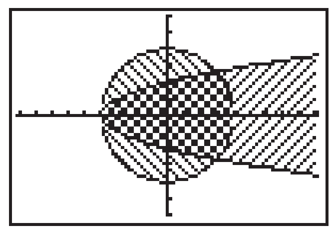
Page 686

1.



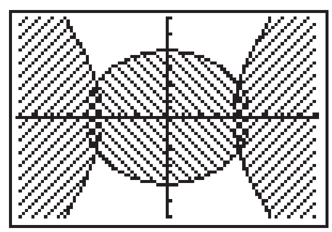
[−9.1, 9.1] scl:1 by [−6, 6] scl:1

2.



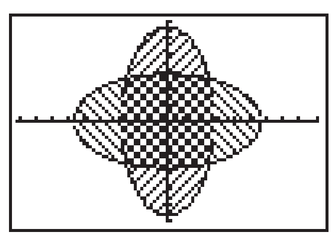
[−9.1, 9.1] scl:1 by [−6, 6] scl:1

3.



[−9.1, 9.1] scl:1 by [−6, 6] scl:1

4.

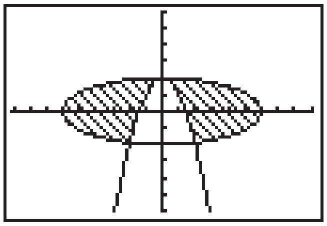


[−9.1, 9.1] scl:1 by [−6, 6] scl:1

5a. 3

5b. Find the points of intersection for the boundary equation by using the TRACE function.

5c. SHADE(−√((36−X²)/9),√((36−X²)/9),−6,−2,3,4);
SHADE(−X²+2,√((36−X²)/9),−2,2,3,4,);
SHADE(−√((36−X²)/9), √((36−X²)/9),2,6,3,4)



$[-9.1, 9.1]$ scl:1 by $[-6, 6]$ scl:1

6. See students' work.

Chapter 10 Study Guide and Assessment

Page 687 Understanding and Using the Vocabulary

1. true
2. false; center
3. false; transverse
4. true
5. false; hyperbola
6. false; axis or axis of symmetry
7. true
8. false; parabola
9. true
10. false; ellipse

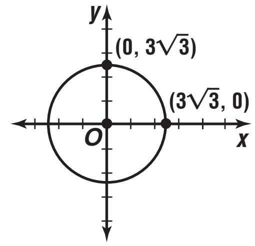
Pages 688–690 Skills and Concepts

11. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(-3 - 1)^2 + [-4 - (-6)]^2}$
$d = \sqrt{20}$ or $2\sqrt{5}$
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1 + (-3)}{2}, \frac{-6 + (-4)}{2}\right)$
$= (-1, -5)$

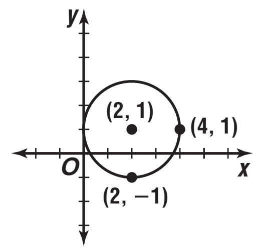
12. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(a + 3 - a)^2 + (b + 4 - b)^2}$
$d = \sqrt{25}$ or 5
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{a + a + 3}{2}, \frac{b + b + 4}{2}\right)$
$= (a + 1.5, b + 2)$

13. $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[3 - (-5)]^2 + [4 - (-2)]^2}$
$= \sqrt{100}$ or 10
$DC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(10 - 2)^2 + [3 - (-3)]^2}$
$= \sqrt{100}$ or 10
$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(10 - 3)^2 + (3 - 4)^2}$
$= \sqrt{50}$ or $5\sqrt{2}$
$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[2 - (-5)]^2 + [-3 - (-2)]^2}$
$= \sqrt{50}$ or $5\sqrt{2}$
Yes; $AB = DC = 10$ and $BC = AD = 5\sqrt{2}$. Since opposite sides of quadrilateral $ABCD$ are congruent, $ABCD$ is a parallelogram.

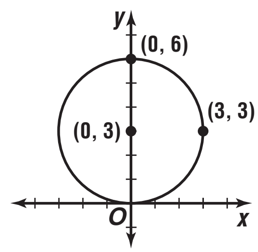
14. $(x - h)^2 + (y - k)^2 = r^2$
$(x - 0)^2 + (y - 0)^2 = \left(3\sqrt{3}\right)^2$
$x^2 + y^2 = 27$



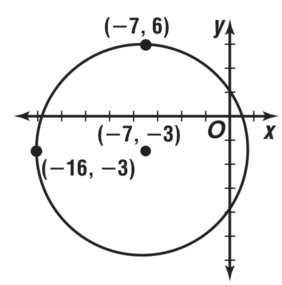
15. $(x - h)^2 + (y - k)^2 = r^2$
$(x - 2)^2 + (y - 1)^2 = 2^2$
$(x - 2)^2 + (y - 1)^2 = 4$



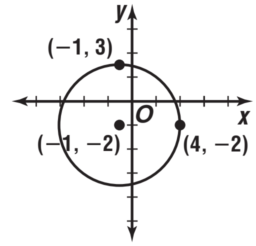
16. $x^2 + y^2 = 6y$
$x^2 + (y^2 - 6y + ?) = 0 + ?$
$x^2 + (y^2 - 6y + 9) = 0 + 9$
$x^2 + (y - 3)^2 = 9$



17. $x^2 + 14x + y^2 + 6y = 23$
$(x^2 + 14x + ?) + (y^2 + 6y + ?) = 23 + ? + ?$
$(x^2 + 14x + 49) + (y^2 + 6y + 9) = 23 + 49 + 9$
$(x + 7)^2 + (y + 3)^2 = 81$



18. $3x^2 + 3y^2 + 6x + 12y - 60 = 0$
$x^2 + y^2 + 2x + 4y - 20 = 0$
$(x^2 + 2x + ?) + (y^2 + 4y + ?) = 20 + ? + ?$
$(x^2 + 2x + 1) + (y^2 + 4y + 4) = 20 + 1 + 4$
$(x + 1)^2 + (y + 2)^2 = 25$



19. $x^2 + y^2 + Dx + Ey + F = 0$

$1^2 + 1^2 + D(1) + E(1) + F = 0 \Rightarrow$
$D + E + F = -2$

$(-2)^2 + 2^2 + D(-2) + E(2) + F = 0 \Rightarrow$
$-2D + 2E + F = -8$

$(-5)^2 + 1^2 + D(-5) + E(1) + F = 0 \Rightarrow$
$-5D + E + F = -26$

$$\begin{array}{l} D + E + F = -2 \\ \underline{(-1)(-5D + E + F) = (-1)(-26)} \\ 6D \qquad\qquad = 24 \\ D = 4 \end{array}$$

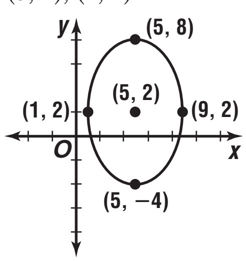
$$\begin{array}{l} -2D + 2E + F = -8 \\ \underline{(-1)(D + \ E + F) = (-1)(-2)} \\ -3D + \ E \qquad = -6 \\ -3(4) + E = -6 \\ E = 6 \end{array}$$

$D + E + F = -2$
$4 + (6) + F = -2$
$F = -12$

$x^2 + y^2 + Dx + Ey - F = 0$
$x^2 + y^2 + 4x + 6y - 12 = 0$
$(x^2 + 4x + ?) + (y^2 + 6y + ?) = 12 + ? + ?$
$(x^2 + 4x + 4) + (y^2 + 6y + 9) = 12 + 4 + 9$
$(x + 2)^2 + (y + 3)^2 = 25$

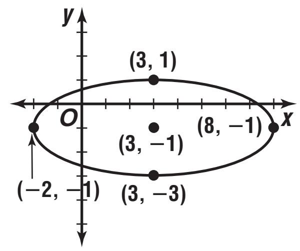
center: $(h, k) = (-2 - 3)$
$r = \sqrt{25}$ or 5

20. center: $(h, k) = (5, 2)$
$a^2 = 36$ $\qquad b^2 = 16$
$a = \sqrt{36}$ or 6 $\qquad b = \sqrt{16}$ or 4
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{6 - 4}$ or $\sqrt{2}$
foci: $(h, k \pm c) = (5, 2 \pm \sqrt{2})$
major axis vertices: $(h, k \pm a) = (5, 2 \pm 6)$ or
$(5, 8), (5, -4)$
minor axis vertices: $(h \pm b, k) = (5 \pm 4, 2)$ or
$(9, 2), (1, 2)$



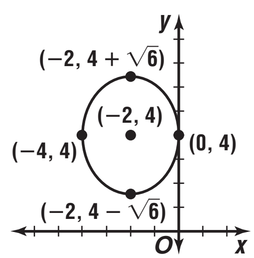
21. $4x^2 + 25y^2 - 24x + 50y = 39$
$4(x^2 - 6x + ?) + 25(y^2 + 2y + ?) = 39 + ? + ?$
$4(x^2 - 6x + 9) + 25(y^2 + 2y + 1) = 39 + 4(9) + 25(1)$
$4(x - 3)^2 + 25(y + 1)^2 = 100$
$\frac{(x-3)^2}{25} + \frac{(y+1)^2}{4} = 1$

center: $(h, k) = (3, -1)$
$a^2 = 25$ $\qquad b^2 = 4$
$a = \sqrt{25}$ or 5 $\qquad b = \sqrt{4}$ or 2
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{25 - 4}$ or $\sqrt{21}$
foci: $(h \pm c, k) = (3 \pm \sqrt{21}, -1)$
major axis vertices: $(h \pm a, k) = (3 \pm 5, -1)$ or
$(8, -1), (-2, -1)$
minor axis vertices: $(h, k \pm b) = (3, -1 \pm 2)$ or
$(3, 1), (3, -3)$



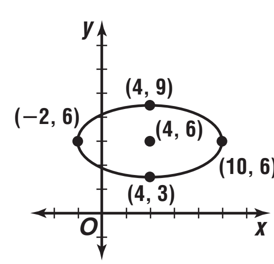
22. $6x^2 + 4y^2 + 24x - 32y + 64 = 0$
$6(x^2 + 4x + ?) + 4(y^2 - 8y + ?) = -64 + ? + ?$
$6(x^2 + 4x + 4) + 4(y^2 - 8y + 16) = -64 + 6(4) + 4(16)$
$6(x + 2)^2 + 4(y - 4)^2 = 24$
$\frac{(x+2)^2}{4} + \frac{(y-4)^2}{6} = 24$

center: $(h, k) = (-2, 4)$
$a^2 = 6$ $\qquad b^2 = 4$
$a = \sqrt{6}$ $\qquad b = \sqrt{4}$ or 2
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{6 - 4}$ or $\sqrt{2}$
foci: $(h, k \pm c) = (-2, 4 \pm \sqrt{2})$
major axis vertices: $(h, k \pm a) = \left(-2, 4 \pm \sqrt{6}\right)$
minor axis vertices: $(h \pm b, k) = (-2 \pm 2, 4)$ or
$(0, 4), (-4, 4)$



23. $x^2 + 4y^2 + 124 = 8x + 48y$
$(x^2 - 8x + ?) + 4(y^2 - 12y + ?) = -124 + ? + ?$
$(x^2 - 8x + 16) + 4(y^2 - 12y + 36) = -124 + 16 + 4(36)$
$(x - 4)^2 + 4(y - 6)^2 = 36$
$\frac{(x-4)^2}{36} + \frac{(y-6)^2}{9} = 36$

center: $(h, k) = (4, 6)$
$a^2 = 36$ $\qquad b^2 = 9$
$a = \sqrt{36}$ or 6 $\qquad b = \sqrt{9}$ or 3
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{36} - 9$ or $3\sqrt{3}$
foci: $(h \pm c, k) = (4 \pm 3\sqrt{3}, 6)$
major axis vertices: $(h \pm a, k) = (4 \pm 6, 6)$ or
$(10, 6), (-2, 6)$
minor axis vertices: $(h, k \pm b) = (4, 6 \pm 3)$ or
$(4, 9), (4, 3)$

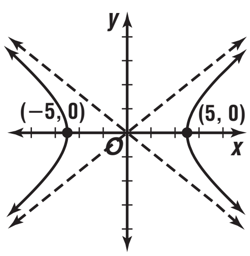


24. $(h, k) = (-4, 1)$
$a = 9$
$b = 6$

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$
$$\frac{(y-1)^2}{9^2} + \frac{[x-(-4)]^2}{6^2} = 1$$
$$\frac{(y-1)^2}{81} + \frac{(x+4)^2}{36} = 1$$

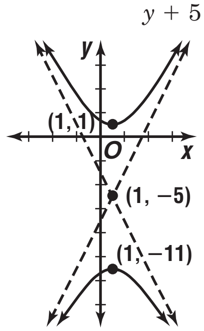
25. center: $(h, k) = (0, 0)$
$a^2 = 25$ $\qquad$ $b^2 = 16$
$a = \sqrt{25}$ or 5 $\qquad$ $b = \sqrt{16}$ or 4
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{25 + 16}$ or $\sqrt{41}$
transverse axis: horizontal
foci: $(h \pm c, k) = (0 \pm \sqrt{41}, 0)$ or $(\pm\sqrt{41}, 0)$
vertices: $(h \pm a, k) = (0 \pm 5, 0)$ or $(-5, 0)$, $(5, 0)$
asymptotes: $y - k = \pm \frac{b}{a}(x - h)$

$$y - 0 = \pm \tfrac{4}{5}(x - 0)$$
$$y = \pm \tfrac{4}{5}x$$



26. center: $(h, k) = (1, -5)$
$a^2 = 36$ $\qquad$ $b^2 = 9$
$a = \sqrt{36}$ or 6 $\qquad$ $b = \sqrt{9}$ or 3
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{36 + 9}$ or $3\sqrt{5}$
transverse axis: vertical
foci: $(h, k \pm c) = \left(1, -5 \pm 3\sqrt{5}\right)$
vertices: $(h, k \pm a) = (1, -5 \pm 6)$ or $(1, 1)$, $(1, -11)$
asymptotes: $y - k = \pm \frac{a}{b}(x - h)$

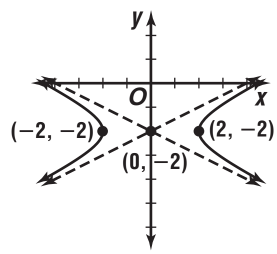
$$y - (-5) = \pm \tfrac{6}{3}(x - 1)$$
$$y + 5 = \pm 2(x - 1)$$



27.
$$x^2 - 4y^2 - 16y = 20$$
$$x^2 - 4(y^2 + 4y + ?) = 20 + ?$$
$$x^2 - 4(y^2 + 4y + 4) = 20 - 4(4)$$
$$x^2 - 4(y + 2)^2 = 4$$
$$\frac{x^2}{4} - \frac{(y+2)^2}{1} = 1$$

center: $(h, k) = (0, -2)$
$a^2 = 4$ $\qquad$ $b^2 = 1$
$a = \sqrt{4}$ or 2 $\qquad$ $b = \sqrt{1}$ or 1
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{4 + 1}$ or $\sqrt{5}$
transverse axis: horizontal
foci: $(h \pm c, k) = (0 \pm \sqrt{5}, -2)$ or $(\pm\sqrt{5}, -2)$
vertices: $(h \pm a, k) = (0 \pm 2, -2)$ or $(-2, -2)$, $(2, -2)$
asymptotes: $y - k = \pm \frac{b}{a}(x - h)$

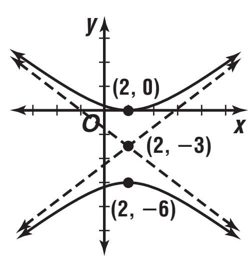
$$y - (-2) = \pm \tfrac{1}{2}(x - 0)$$
$$y + 2 = \pm \tfrac{1}{2}x$$



28.
$$9x^2 - 16y^2 - 36x - 96y + 36 = 0$$
$$9(x^2 - 4x + ?) - 16(y^2 + 6y + ?) = -36 + ? + ?$$
$$9(x^2 - 4x + 4) - 16(y^2 + 6y + 9) = -36 + 9(4) - 16(9)$$
$$9(x - 2)^2 - 16(y + 3)^2 = -144$$
$$\frac{(y+3)^2}{9} - \frac{(x-2)^2}{16} = 1$$

center: $(h, k) = (2, -3)$
$a^2 = 9$ $\qquad$ $b^2 = 16$
$a = \sqrt{9}$ or 3 $\qquad$ $b = \sqrt{16}$ or 4
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{9 + 16}$ or 5
transverse axis: vertical
foci: $(h, k \pm c) = (2, -3 \pm 5)$ or $(2, 2)$, $(2, -8)$
vertices: $(h, k \pm a) = (2, -3 \pm 3)$ or $(2, 0)$, $(2, -6)$
asymptotes: $y - k = \pm \frac{a}{b}(x - h)$

$$y - (-3) = \pm \tfrac{3}{4}(x - 2)$$
$$y + 3 = \pm \tfrac{3}{4}(x - 2)$$



29. $c = 9$

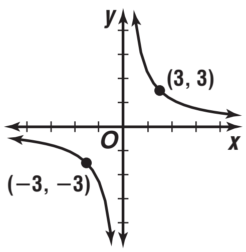
quadrants: I and III

transverse axis: $y = x$

vertices: $xy = 9$ $\quad$ $xy = 9$

$3(3) = 9$ $\quad$ $(-3)(-3) = 9$

$(3, 3)$ $\quad$ $(-3, -3)$



30. $2b = 10$

$b = 5$

center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1 + 1}{2}, \frac{-1 + 5}{2}\right)$

$= (1, 2)$

transverse axis: vertical

a = distance from center to a vertex

$= |2 - (-1)|$ or 3

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$

$$\frac{(y - 2)^2}{3^2} - \frac{(x - 1)^2}{5^2} = 1$$

$$\frac{(y - 2)^2}{9} - \frac{(x - 1)^2}{25} = 1$$

31. center: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 6}{2}, \frac{-3 + (-3)}{2}\right)$

$= (2, -3)$

a = distance from center to a vertex

$= |2 - (-2)|$ or 4

c = distance from center to a focus

$= |2 - (-4)|$ or 6

$b^2 = c^2 - a^2$

$b^2 = 6^2 - 4^2$

$b^2 = 20$

transverse axis: horizontal

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

$$\frac{(x - 2)^2}{4^2} - \frac{[y - (-3)]^2}{20} = 1$$

$$\frac{(x - 2)^2}{16} - \frac{(y + 3)^2}{20} = 1$$

32. vertex: $(h, k) = (5, 3)$

$4p = 8$

$p = 2$

focus $(h, k + p) = (5, 3 + 2)$ or $(5, 5)$

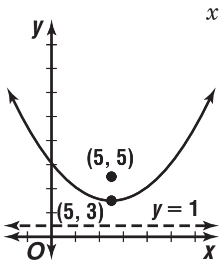
directrix: $y = k - p$

$y = 3 - 2$

$y = 1$

axis of symmetry: $x = h$

$x = 5$



33. vertex: $(h, k) = (1, -2)$

$4p = -16$

$p = -4$

focus: $(h + p, k) = (1 + (-4), -2)$ or $(-3, -2)$

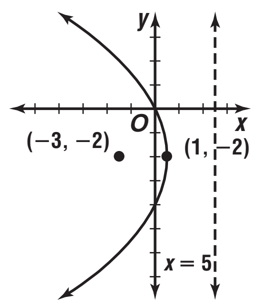
directrix: $x = h - p$

$x = 1 - (-4)$

$x = 5$

axis of symmetry: $y = k$

$y = -2$



34. $y^2 + 6y - 4x = -25$

$y^2 + 6y + ? = 4x - 25 + ?$

$y^2 + 6y + 9 = 4x - 25 + 9$

$(y + 3)^2 = 4(x - 4)$

vertex: $(h, k) = (4, -3)$

$4p = 4$

$p = 1$

focus: $(h + p, k) = (4 + 1, -3)$ or $(5, -3)$

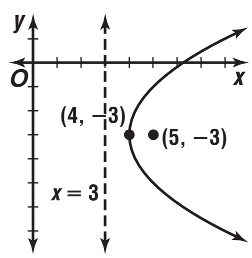
directrix: $x = h - p$

$x = 4 - 1$

$x = 3$

axis of symmetry: $y = k$

$y = -3$



35. $x^2 + 4x = y - 8$

$x^2 + 4x + 4 = y - 8 + 4$

$(x + 2)^2 = y - 4$

vertex: $(h, k) = (-2, 4)$

$4p = 1$

$p = \frac{1}{4}$

focus: $(h, k + p) = \left(-2, 4 + \frac{1}{4}\right)$ or $(-2, 4.25)$

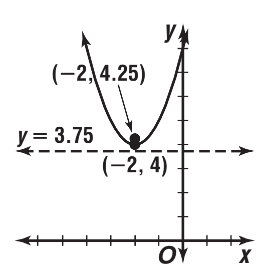
directrix: $y = k - p$

$y = 4 - \frac{1}{4}$

$y = 3.75$

axis of symmetry: $x = h$

$x = -2$



36. vertex: $(h, k) = (-1, 3)$

$(y - k)^2 = 4p(x - h)$

$(7 - 3)^2 = 4p[-3 - (-1)]$

$16 = 8p$

$2 = p$

Since parabola opens left, $p = -2$.

$(y - k)^2 = 4p(x - h)$

$(y - 3)^2 = 4(-2)[x - (-1)]$

$(y - 3)^2 = -8(x + 1)$

37. vertex: $(h, k) = \left(5, \frac{2-4}{2}\right)$ or $(5, -1)$

focus: $(h, k + p) = (5, 2)$

$k + p = 2$

$-1 + p = 2$

$p = 3$

$(x - h)^2 = 4p(y - k)$

$(x - 5)^2 = 4(3)[y - (-1)]$

$(x - 5)^2 = 12(y + 1)$

38. $A = 5$, $c = 2$; ellipse

39. $A = C = 0$; equilateral hyperbola

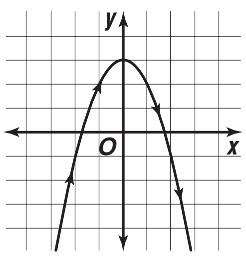
40. $A = C = 5$; circle

41. $C = 0$; parabola

42. $y = -t^2 + 3$

$y = -x^2 + 3$

t	x	y	(x, y)
-2	-2	-1	$(-2, -1)$
-1	-1	2	$(-1, 2)$
0	0	3	$(0, 3)$
1	1	2	$(1, 2)$
2	2	-1	$(2, -1)$

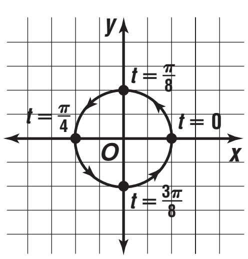


43. $x = \cos 4t$ $\quad$ $y = \sin 4t$

$\cos^2 4t + \sin^2 4t = 1$

$x^2 + y^2 = 1$

t	x	y	(x, y)
0	1	0	$(1, 0)$
$\frac{\pi}{8}$	0	1	$(0, 1)$
$\frac{\pi}{4}$	-1	0	$(-1, 0)$
$\frac{3\pi}{8}$	0	-1	$(0, -1)$



44. $x = 2 \sin t$ $\quad$ $y = 3 \cos t$

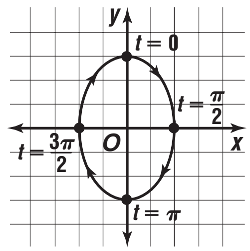
$\frac{x}{2} = \sin t$ $\quad$ $\frac{y}{3} = \cos t$

$\sin^2 t + \cos^2 t = 1$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

t	x	y	(x, y)
0	0	3	$(0, 3)$
$\frac{\pi}{2}$	2	0	$(2, 0)$
π	0	-3	$(0, -3)$
$\frac{3\pi}{2}$	-2	0	$(-2, 0)$



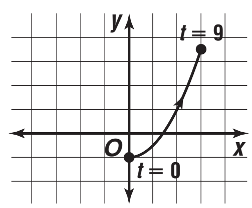
45. $x = \sqrt{t}$

$x^2 = t$

$y = \frac{t}{2} - 1$

$y = \frac{x^2}{2} - 1$

t	x	y	(x, y)
0	0	-1	$(0, -1)$
4	2	1	$(2, 1)$
9	3	3.5	$(3, 3.5)$



46. Sample answer:

Let $x = t$.

$y = 2x^2 + 4$

$y = 2t^2 + 4, -\infty < t < \infty$

47. Sample answer:

$x^2 + y^2 = 49$

$\frac{x^2}{49} + \frac{y^2}{49} = 1$

$\left(\frac{x}{7}\right)^2 + \left(\frac{y}{7}\right)^2 = 1$

$\sin^2 t + \cos^2 t = 1$

$\left(\frac{x}{7}\right)^2 = \sin^2 t$ $\quad$ $\left(\frac{y}{7}\right)^2 = \cos^2 t$

$\frac{x}{7} = \sin t$ $\quad$ $\frac{y}{7} = \cos t$

$x = 7 \sin t$ $\quad$ $y = 7 \cos t, 0 \leq t \leq 2\pi$

48. Sample answer:

$$\frac{x^2}{36} + \frac{y^2}{81} = 1$$
$$\left(\frac{x}{6}\right)^2 + \left(\frac{y}{9}\right)^2 = 1$$
$$\cos^2 t + \sin^2 t = 1$$
$$\left(\frac{x}{6}\right)^2 = \cos^2 t \qquad \left(\frac{y}{9}\right)^2 = \sin^2 t$$
$$\frac{x}{6} = \cos t \qquad \frac{y}{9} = \sin t$$
$$x = 6\cos t \qquad y = 9\sin t,\ 0 \le t \le 2\pi$$

49. Sample answer:

Let $y = t$.

$x = -y^2$

$x = -t^2,\ -\infty < t < \infty$

50. $B^2 - 4AC = 0 - 4(4)(9)$

$= -144$

$A \neq C$; ellipse

$$4x^2 + 9y^2 = 36$$
$$4\left(x'\cos\frac{\pi}{6} + y'\sin\frac{\pi}{6}\right)^2 + 9\left(-x'\sin\frac{\pi}{6} + y'\cos\frac{\pi}{6}\right)^2 = 36$$
$$4\left(\frac{\sqrt{3}}{2}x' + \frac{1}{2}y'\right)^2 + 9\left(-\frac{1}{2}x' + \frac{\sqrt{3}}{2}y'\right)^2 = 36$$
$$4\left[\frac{3}{4}(x')^2 + \frac{\sqrt{3}}{2}x'y' + \frac{1}{4}(y')^2\right]$$
$$+ 9\left[\frac{1}{4}(x')^3 - \frac{\sqrt{3}}{2}x'y' + \frac{3}{4}(y')^2\right] = 36$$
$$3(x')^2 + 2\sqrt{3}x'y' + (y')^2$$
$$+ \frac{9}{4}(x')^2 - \frac{9\sqrt{3}}{2}x'y' + \frac{27}{4}(y')^2 = 36$$
$$\frac{21}{4}(x')^2 - \frac{5\sqrt{3}}{2}x'y' + \frac{31}{4}(y')^2 = 36$$
$$21(x')^2 - 10\sqrt{3}x'y' + 31(y')^2 - 144 = 0$$

51. $B^2 - 4AC = 0 - 4(0)(1)$

$= 0$

parabola

$$y^2 - 4x = 0$$
$$(-x'\sin 45° + y'\cos 45°)^2 - 4(x'\cos 45° + y'\sin 45°) = 0$$
$$\left(-\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right)^2 - 4\left(\frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y'\right) = 0$$
$$\frac{1}{2}(x')^2 - x'y' + \frac{1}{2}(y')^2 - 2\sqrt{2}x' + 2\sqrt{2}y' = 0$$
$$(x')^2 - 2x'y' + (y')^2 - 4\sqrt{2}x' + 4\sqrt{2}y' = 0$$

52. $B^2 - 4AC = 0 - 4(4)(-16)$

$= 256$

hyperbola

$$4x^2 - 16(y-1)^2 = 64$$
$$4(x-h)^2 - 16(y-k-1)^2 = 64$$
$$4(x-1)^2 - 16(y+2-1)^2 = 64$$
$$4(x-1)^2 - 16(y+1)^2 = 64$$
$$4(x^2 - 2x + 1) - 16(y^2 + 2y + 1) = 64$$
$$4x^2 - 8x + 4 - 16y^2 - 32y - 16 - 64 = 0$$
$$x^2 - 4y^2 - 2x - 8y - 19 = 0$$

53. $B^2 - 4AC = \left(2\sqrt{3}\right)^2 - 4(6)(8)$

$= -180$

$A \neq C$; ellipse

$\tan 2\theta = \frac{B}{A - C}$

$\tan 2\theta = \frac{2\sqrt{3}}{6 - 8}$

$\tan 2\theta = -\sqrt{3}$

$2\theta = -60°$

$\theta = -30°$

54. $B^2 - 4AC = (-6)^2 - 4(1)(9)$

$= 0$

parabola

$\tan 2\theta = \frac{B}{A - C}$

$\tan 2\theta = \frac{-6}{1 - 9}$

$\tan 2\theta = \frac{3}{4}$

$2\theta \approx 36.86989765°$

$\theta \approx 18°$

55. $(x-1)^2 + 4(y-1)^2 = 20$

$(y-1)^2 + 4(y-1)^2 = 20$

$5(y-1)^2 = 20$

$(y-1)^2 = 4$

$y - 1 = \pm 2$

$y = 3$ or -1

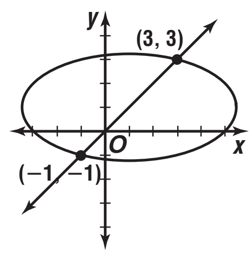
$y = 3 \qquad x = y$

$\qquad x = 3$

$y = -1 \qquad x = y$

$\qquad x = -1$

$(3, 3), (-1, -1)$



56. $2x - y = 0$

$2x = y$

$y^2 = 49 + x^2$

$(2x)^2 = 49 + x^2$

$4x^2 = 49 + x^2$

$3x^2 = 49$

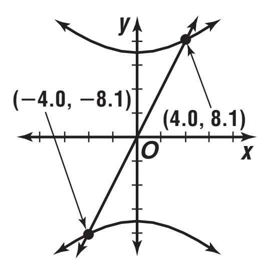
$x \approx \pm 4.04$

$2x - y = 0 \qquad 2x - y = 0$

$2(4.04) - y \approx 0 \qquad 2(-4.04) - y \approx 0$

$y \approx 8.08 \qquad y \approx -8.08$

$(4.0, 8.1), (-4.0, -8.1)$



57. $x^2 - 4x - 4y = 4$

$x^2 - 4x - 4 = 4y$

$(x - 2)^2 + 4y = 0$

$(x - 2)^2 + x^2 - 4x - 4 = 0$

$x^2 - 4x + 4 + x^2 - 4x - 4 = 0$

$2x^2 - 8x = 0$

$2x(x - 4) = 0$

$2x = 0 \qquad x - 4 = 0$

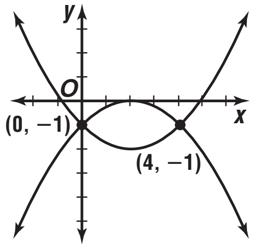
$x = 0 \qquad x = 4$

$(x - 2)^2 + 4y = 0 \qquad (x - 2)^2 + 4y = 0$

$(0 - 2)^2 + 4y = 0 \qquad (4 - 2)^2 + 4y = 0$

$y = -1 \qquad y = -1$

$(0, -1), (4, -1)$



58. $xy = -4$

$y = -\frac{4}{x}$

$x^2 + y^2 = 12$

$x^2 + \left(-\frac{4}{x}\right)^2 = 12$

$x^2 + \frac{16}{x^2} = 12$

$x^4 + 16 - 12x^2 = 0$

$(x^2)^2 - 12(x^2) + 16 = 0$

$x^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x^2 = \frac{12 \pm \sqrt{12^2 - 4(1)(16)}}{2(1)}$

$x^2 \approx 10.472$ or $x^2 \approx 1.528$

$x \approx \pm\, 3.236 \qquad x \approx 1.236$

$xy = -4 \qquad xy = -4$

$3.236y \approx -4 \qquad 1.236y \approx -4$

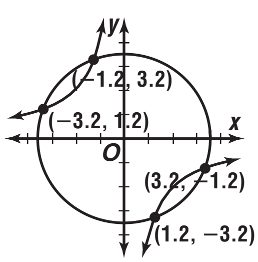
$y \approx -1.236 \qquad y \approx -3.236$

$xy = -4 \qquad xy = -4$

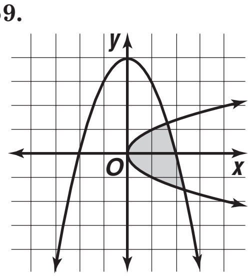
$-3.236y \approx -4 \qquad -1.236y \approx -4$

$y \approx 1.236 \qquad y \approx 3.236$

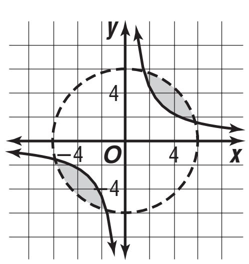
$(3.2, -1.2), (-3.2, 1.2), (1.2, -3.2), (-1.2, 3.2)$



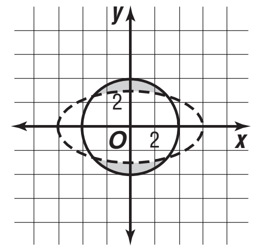
59.



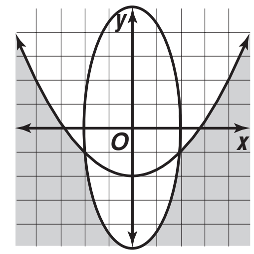
60.



61.



62.



Page 691 Applications and Problem Solving

63a. $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$r = \sqrt{(12 - 0)^2 + (16 - 0)^2}$

$r = 20$

$x^2 + y^2 = r^2$

$x^2 + y^2 = 20^2$

$x^2 + y^2 = 400$

63b. area of watered portion $= \pi r^2$

$= \pi 20^2$

$\approx 1256.6 \text{ ft}^2$

area of backyard $= \ell\omega$

$= 50(40)$

$= 2000 \text{ ft}^2$

area of nonwatered portion $\approx 2000 - 1256.6$

$\approx 743.4 \text{ ft}^2$

percent not watered $\approx \frac{743.4}{2000}$

≈ 0.37

about 37%

64. $2a = 2{,}000 \qquad e = \frac{c}{a}$

$a = 6000 \qquad 0.2 = \frac{c}{6000}$

$1200 = c$

$b^2 = a^2 - c^2$

$b^2 = 6000^2 - 1200^2$

$b^2 = 34{,}560{,}000$

$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$

$\frac{(x - 0)^2}{6000^2} + \frac{(y - 0)^2}{34{,}560{,}000} = 1$

$\frac{x^2}{36{,}000{,}000} + \frac{y^2}{34{,}560{,}000} = 1$

65. $a = 3.5$

$b = 3$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{3.5^2 - 3^2}$

$c \approx 1.8$

about 1.8 feet from the center

Page 691 Open-Ended Assessment

1. Sample answer:

$e = \frac{c}{a}$

$\frac{1}{9} = \frac{c}{a}$

Let $a = 9$.

$\frac{1}{9} = \frac{c}{9}$

$c = 1$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{9^2} + \frac{y^2}{80} = 1$

$\frac{x^2}{81} + \frac{y^2}{80} = 1$

$b^2 = a^2 - c^2$

$b^2 = 9^2 - 1^2$

$b^2 = 80$

2. Sample answer:

axis of symmetry: $x = h$

$x = 2$, so $h = 2$

focus: $(h, k + p) = (2, 5)$

$k + p = 5$

Let $k = 2$, $p = 3$.

$(x - h)^2 = 4p(y - k)$

$(x - 2)^2 = 4(3)(y - 2)$

$(x - 2)^2 = 12(y - 2)$

SAT & ACT Preparation

Page 693 SAT & ACT Practice

1. Add the two numbers of parts to get the whole, 8. The fraction of red jelly beans to the whole is $\frac{3}{8}$. The total number of jelly beans is 160. The number of red jelly beans is $\frac{3}{8}(160)$ or 60. The correct choice is C.
Or you can use a ratio box. Multiply by 20.

Green	Red	Whole
5	3	8
	60	160

2. Notice the capitalized word *EXCEPT.* You might want to try the plug-in method on this problem. Choose a value for b that is an odd integer, say 1. Then substitute that value for b in the equation.

$a^2b = 12^2$

$a^2(1) = 12^2$

$a^2 = 12^2$

$a = 12$

Check the answer choices for divisors of this value of a. 12 is divisible by 3, 4, 6, and 12, but not by 9. The correct choice is D.

3. The information in the question confirms the information given in the figure. Recall the formula for the area of a triangle — one half the base times the height. The triangle DCB is obtuse, so the height will lie outside of the triangle.
Let $\overline{DC}$ be the base. The length of the base is 6. The height will be *equal* to 7, since it is a line segment parallel to $\overline{AD}$ through point B.

$A = \frac{1}{2}bh$

$= \frac{1}{2}(6)(7)$ or 21

The correct choice is A.

4. The problem asks how many more girls there are than boys. First find how many girls and how many boys there are in the class.
One method is to find the fraction of girls in the whole class and the fraction of boys in the whole class. Since the ratio of girls to boys is 4 to 3, the fraction of girls in the whole class is $\frac{4}{7}$. Find the number of girls in the class by multiplying this fraction by 35.

$\frac{4}{7}(35) = 20$ There are 20 girls in the class.

Using the same process, the fraction of boys is $\frac{3}{7}$.

$\frac{3}{7}(35) = 15$ There are 15 boys in the class.

So there are 5 more girls than boys. The correct choice is D.
Another method is to use a "Ratio Box." First enter the given information, shown in the darker cells below. Then enter the number for the total of the first row, 7. To go from the total of 7 to the total of 35, you must multiply by 5. Write a 5 in each cell in the second row.

Girls	**Boys**	**Total**
4	3	7
×5	×5	×5
20	15	35

Then multiply the two numbers in the first column to get 20 girls, shown with a dark border. Multiply the second column to get 15 boys. Subtract to find there are 5 more girls than boys.

5. Set A is the set of all positive integers less than 30. Set B is the set of all positive multiples of 5. The intersection of Sets A and B is the set of all elements that are in both Set A and Set B. The intersection consists of all positive multiples of 5 which are also less than 30. The intersection of the two sets is {5, 10, 15, 20, 25}.
The correct choice is A.

6. For a quadratic equation in the form $y = a(x - h)^2 + k$, the coordinates of the vertex of the graph of the function are given by the ordered pair (h, k). So the vertex of the graph of $y = \frac{1}{2}(x - 3)^2 + 4$ has coordinates (3, 4).
The correct choice is C.

7. On the SAT, if you forget the relationships for 45° right triangles, look at the Reference Information in the gray box at the beginning of each mathematics section of the exam. The measure of each leg of a 45–45–90 triangle is equal to the length of the hypotenuse divided by $\sqrt{2}$. Multiply both numerator and denominator by $\sqrt{2}$ and simplify.

$$BC = \frac{8}{\sqrt{2}}$$
$$= \frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{8\sqrt{2}}{2} \text{ or } 4\sqrt{2}$$

The correct choice is D. You could also use the Pythagorean Theorem and the fact that the two legs must be equal in length, but that method might take more time.

8. Form a ratio using the given fractions as numerator and denominator. Write a proportion, using x as the unknown. Multiply the cross-products. Solve for x.

$$\frac{\frac{1}{7}}{\frac{1}{5}} = \frac{100}{x}$$
$$\frac{1}{7}x = \frac{1}{5}(100)$$
$$\frac{1}{7}x = 20$$
$$x = 140$$

The correct choice is E.

9. Let d represent the number of dimes in the jar. Since there are 4 more nickels than dimes, there are $d + 4$ nickels in the jar. So, the ratio of dimes to nickles in the jar is $\frac{d}{d + 4}$. This ratio is less than 1. The only answer choice that is less than 1 is choice A, $\frac{8}{10}$. If $\frac{d}{d + 4} = \frac{8}{10}$, then $d = 16$. So, there are 16 dimes and 16 + 4 or 20 nickels in the jar, and $\frac{16}{20} = \frac{8}{10}$.
The correct choice is A.

10. Set up a proportion.

$$\frac{\text{total liters}}{\text{total bottles}} = \frac{x \text{ liters}}{1 \text{ bottle}}$$
$$\frac{8}{20} = \frac{x}{1}$$
$$20x = 8$$
$$x = 0.4 \text{ or } \frac{2}{5}$$

The correct answer is .4 or 2/5.